TÉCNICO LISBOA

A comparative analysis on efficiency using slacks-based DEA, standard DEA versus network DEA.

The case study of José de Mello Saúde.

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Thesis to obtain the Master of Science Degree in

Biomedical Engineering

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December 2020

Declaration

I declare that this document is an original work of my own authorship and that it fulfills all the requirements of the Code of Conduct and Good Practices of the Universidade de Lisboa.

Preface

The work presented in this dissertation was performed at CEG-IST Center of Management Studies of Instituto Superior Técnico, and CERIS during the period from February to December of 2020, under the ´ supervision of Professor Diogo da Cunha Ferreira and Professor José Rui Figueira, and within the frame of the hSNS FCT - Research Project (PTDC/EGEOGE/30546/2017): Portuguese public hospital performance assessment using a multicriteria decision analysis framework. The thesis was co-supervised at CUF by João Leal.

Acknowledgments

The completion of the present dissertation was only possible thanks to the assistance and support of a group of individuals.

First of all, I would like to express my gratitude towards my family as a whole, for being a source of strength during my entire life, and each member in its own personal style, for the inestimable life lessons. In particular, to my grandparents for being my biggest fans, to my mother for encouraging me to become my best version, to my father for his well-intended pickiness that allows me to pursue my dreams, and to my sister for being my incredible cheerleader.

Undoubtedly, I would also like to recognize the priceless assistance of my supervisors, Professor Diogo da Cunha Ferreira and Professor José Rui Figueira, for their guidance and knowledge share in moments of blur vision. Not forgetting my irreplaceable external supervisor, João Paulo Leal, which was a crucial piece in the construction this thesis. Without him, the following dissertation wouldn't be possible.

Last but not least, to all my friends and colleagues that helped me grow as a person and were always there for me during the good and bad times in college life. In particular, to Miguel Pereira for his help and support in enlighten me through my inexperience.

To each and every one of you – Thank you.

Abstract

The health care sector is a complex system which involves not only the health provider organizations but also the surrounding environment. In specific, hospitals are influenced by external (national health and financial environment) and internal (staff productivity and structure fluxes) variables. Therefore, new forms to evaluate hospital's efficiency, especially the ones which incorporate services' interconnections, enhance the health sector value. Hence, the development of network DEA methodology capable of being applied in any hospital is the first step to uncover more sophisticated techniques. Accordingly, this study proposes a comparison between the most used methodology for performance evaluation, DEA, and the most recent improvement, network DEA. By taking into consideration the current growth of private hospitals, the efficiency analysis was applied to the largest private healthcare provider in Portugal, CUF. Therefore, the present dissertation is filling two knowledge gaps, lack of academic efficient analysis in the private sector and the absence of comparative studies between standard DEA and network DEA. The obtained results for the new model revealed an increase in the number of efficient hospital's services units, as well as a general decrease in the number of global efficient hospital units. Finally, by crossing information with the business viewpoint, the inclusion of hospital fluxes is enlightened. The network DEA provides a holistic view of hospital dynamics.

Keywords

Efficiency; Methodologies' comparison; Network DEA; Private sector;

Resumo

O setor de saúde é um sistema complexo que envolve não só organizações provedoras de saúde, mas também o meio envolvente. Especificamente, os hospitais são influenciados por variáveis externas (saúde nacional e ambiente financeiro) e internas (eficiência do pessoal e fluxos estruturais). Assim, novas formas para avaliar a eficiência hospitalar, em especial aquelas que consideram as conexões internas, aprimoram o valor do sector de saúde. Portanto, a criação de uma metodologia network DEA capaz de ser aplicada a qualquer hospital é o primeiro passo para desenvolver técnicas mais sofisticadas. Nesse sentido, este trabalho, propõe uma comparação entre a metodologia de avaliação de eficiência mais utilizada, DEA, e a mais recente melhoria, network DEA. Considerando também o crescimento atual do sector privado, a análise foi aplicada à maior organização privada prestadora de saúde em Portugal, a CUF. Desse modo, esta dissertação dá resposta a duas lacunas de conhecimento, a falta de análises académicas em eficiência no setor privados e a ausência de estudos comparativos entre standard DEA e network DEA. Os resultados obtidos para o novo modelo revelaram um aumento no número de unidades eficientes de serviços hospitalares, bem como uma diminuição geral do número de unidades hospitalares globalmente eficientes. Por fim, ao cruzar a informação com a visão empresarial, a inclusão dos fluxos hospitalares é enaltecida. O network DEA fornece uma visão holística da dinâmica hospitalar.

Palavras Chave

Eficiência; Comparação de metodologias; Network DEA; Setor privado

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Acronyms

AE	Allocative efficiency
BB	Black-box
BI	Business Intelligence
CRS	Constant Returns to Scale
DEA	Data Envelopment Analysis
DMU	Decision making unit
DN-DEA	Dynamic-Network DEA
GDP	Gross domestic produc
HIT	Health Information Technology
IM	Information management
LSR	Least-Squares Regression
NIRS	Non-Increasing Returns-to-Scale
OECD	Organization for Economic Co-operation and Development
OLS	Ordinary Least Squares
SDGs	Sustainable Development Goals
SFA	Stochastic Frontier Analysis
TE	Technical efficiency
UHC	Universal health coverage
VHI	Voluntary health insurance
VRS	Variable Returns to Scale
WHO	World Health Organization

1

Introduction

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This chapter consists of four sections. The first section explains the motivation, pointing the value of the problem dealt in this dissertation. The second section clarifies the objectives to be achieved with this study. The third section presents the methodology to address the presented problem. The fourth and last section outlines the dissertation's structure.

1.1 Context and motivation

Efficiency, independently from the sector it is applied, is an important factor in the regulation of the resources used and produced (Curristine et al., 2007; Kao, 2017).[1] Being on a period when waste is economically, environmentally, and socially undesired, evaluating efficiency is crucial (Ekins and Hughes, 2016).

In specific for the health sector, where we are dealing with a fundamental human right, as envisage by World Health Organization (WHO) constitution (1946), "*... the highest attainable standard of health as a fundamental right of every human being.*", it is essential to improve the quality of delivered care. The population health is highly related to the hospital's efficiency: when one increases, the other is also likely to increase (Mossialos and Grand, 2019).[2] Therefore, inefficiency in the health system can result in the deny of treatment and health improvement to patients, due to the excessive consumption of resources. Otherwise, patients would have received treatment. However, that is not the only sacrificed sector, education or nutrition will also perceive the system's inefficiencies. Moreover, it compromises the willingness of the society to contribute to the funding of health services, thereby harming social solidarity, health system performance, and social welfare (Cylus et al., 2016b).

For instance, universal coverage requires a high level of health service output, access for everyone in need to a core set of health interventions (Footnote 2). In Portugal, where this study is conducted, the Portuguese State declared, in the article 64 of the Decree-Law no. 413/71, "*everyone has the right to health protection and the duty to defend and promote it*". Onward, in the same law decree is enacted that the health ministry should "*support the administration of hospital establishments under the responsibility of the General Directorate and act technically with similar private establishments, providing assistance to all, in order to promote the efficiency and services' economy*".

Nevertheless, the elaboration of a strategy based on complete measurements have not been executed, as supported by an WHO study.[3] That Report stated that Portugal is one of only four countries (of 33 analysed) that reduced public health expenditure between 2000 and 2017 (The Lancet., 2019). In

[1] O'Brien, M., Fischer, S., Schepelmann, P., and Bringezu, S. (2012).Resource Efficiency in EuropeanIndustry. European Parliament. Retrieved from https://www.europarl.europa.eu/RegData/etudes/etudes/join/2012/492457/IPOL-ITRE_ET(2012)492457_EN.pdf

[2] Chisholm, D. and Evans, D. B. (2010). Improving health system efficiency as a means of moving to-wards universal coverage. In World Health Report: Background Paper, number 28. World Health Organization

[3] WHO. (2019).Healthy, prosperous lives for all:the european health equity status re-port. Technical report. Retreived from http://www.euro.who.int/en/publications/abstracts/health-equity-status-report-2019 [Accessed: February 28, 2020]

simultaneous, the economic recession and fiscal consolidation measures reduced spending by nearly one percentage point, from 9.8% to 9% of GDP in 2015, compared with the EU average of 9.9% (OECD, 2017). As this falling investment is preventing the modernisation of hospitals and replacement of obsolete medical equipment, private care is expanding (The Lancet., 2019). Overall, in 2014, about 10 847 deaths were deemed to be avoidable through the delivery of higher quality and more timely health. As a consequence of these factors, private Voluntary health insurance (VHI) has been growing over the years. In general in the EU, since the percentage is equivalent, being at 5% of the health financing (OECD, 2017). Accordingly, two major factors made private hospitals appear as a good option to provide the data for the performance analysis in this study.

1. The increase of the private sector in the health domain, which is not only observed in Portugal but worldwide (Kruse et al., 2018).

2. The data availability, which is lower in the public sector, making it harder to analyse public hospitals efficiency (Barr, 2007; Evans et al., 2019). However, in private hospitals, since financial return is one major factor, the data is largely available.

In the last three decades, efficiency analysis in health sector has been mainly performed using "black-box" models, such as Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA). Therefore the hospital is considered as a whole, with global input and output variables, ignoring internal interactions (Hollingsworth, 2016).[4] Over 400 published applications have used these methods (Hollingsworth, 2003, 2008; Hollingsworth and Scott, 2012). However, DEA (Charnes et al., 1978; Banker et al., 1984; Charnes et al., 1994) has been applied in more than 90% of health care setting, since it can account for multiple inputs and outputs, varying weights and returns to scale (Hollingsworth, 2016; Kao, 2017).

Nevertheless, since there are several services (divisions) in one single hospital, inefficiency problems can then arise from relations that are not being considered. Therefore, the process of identifying the source of inefficiency is potentially skewed, resulting in inappropriate correction, and possible waste of more resources. For instance, inefficiency can result from a particular service not treating inputs/outputs correctly or from an incorrect flow between services. As for that, it is essential to assess these service interactions to evaluate hospital performance. Network DEA (Färe and Grosskopf, 2000; Kao, 2017) makes room to uncover the potential of innovative and more specific efficiency methodologies, which may facilitate the setting of benchmarks on funding specific services or global hospitals.

Hopefully this will result in a methodology's improvement for efficiency analysis in the health sector worldwide. Specifically in Portugal, which has been recognized for lack of long-term strategy and bad resource allocation. Therefore, our sincere wish is to contribute to a better Portuguese future, in particular for the private health sector.

[4] "Tools and Methodologies to assess the efficiency of health care services in Europe", 2019. European Union's report.

1.2 Objectives

This dissertation aims to compare two performance methodologies by evaluating the efficiency of a specified group of private hospitals. Considering that efficiency is not only depending on the global variables of a hospital, but also on the intermediate variables associated with the different services. It is necessary to study not only the input and output variables of the global hospital but also the ones of each particular service. Thus, to assess efficiency over its full extent, this study is applying a network DEA model as an innovative methodology. The model allows the evaluation of variables flow between services, resulting in a study of the global and the partial efficiency of each hospital, in an attempt of identifying efficiency and inefficiency foci.

Initially, the three most active CUF's hospitals will be evaluated using standard DEA. Each one will be analysed over the three previous years (2017, 2018, 2019), dividing the data into months (12 months x 3 year = 36 units of analysis for each Hospital). However, in this stage, the model will only use the global variables without considering the services, as has been done in multiple studies for other hospitals. Then, the following approaches enhance the differentiation of this dissertation. The three major hospital's services will be evaluated using standard DEA, as they were independent from the rest of the hospital. Resulting in the determination of its efficiency score, as a whole, instead of the common analysis of the global hospital. Nonetheless, as previously mentioned, one hospital may be efficient when there are inefficient services or inefficient while every division is efficient independently. Based on that, another model is applied. The network DEA allows the identification of these efficiency flows, more information about the practises in each service and its influence for the global efficiency. After the efficiency analysis has been processed, a standard point of reference will be set, also named benchmark, allowing the comparison between different management moments. It will also facilitate the decision-making for the managers of private health care entities, since information about good or bad practises from the past activities will be provided.

Although the main objective of this thesis is to compare standard and network DEA and conclude which one is best technique, other objectives are sintetized as follows:

1. To understand the particular case of the hospital organization in study, as well as its story and evolution on the topic of efficiency analysis;

2. To conceive the DEA model for the studied hospitals and each one of their services, and the network DEA model for the service interactions and global hospitals;

3. To describe each model and the obtained results for both methodologies;

4. To identify the interdepartmental (in)efficiencies in a set of private health care providers;

5. To help on designing new studies, where this evaluation could have major impact.

1.3 Methodology

The methodology chosen to approach the present dissertation consists of five stages, as schematized in the Figure 1.3.1. The first stage defines and describes the problem, as well as its contextualization in the health sector, particularly in the analysed private hospitals. For the second stage, a literature review is required for a better understanding of the problem, to confer a solid theoretical basis and to analyse previous solutions. The third stage is the creation of the models necessary for the evaluation of the proposed study. Therefore, in this phase is necessary to collect and process the data set, after defining the input and output variables for each division of the model. The fourth stage is the application of the model, in other words, the evaluation of the proposed problem with the built model to produce results. For the fifth and last stage, one discusses and analyses the obtained results, withdraw conclusions on the problem presented at the beginning and suggest future research to be developed.

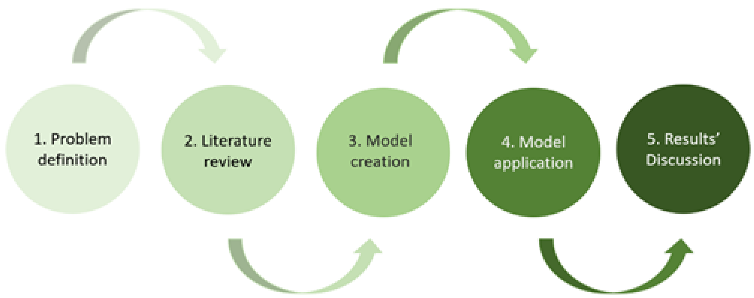

Figure 1.3.1: Research methodological sequential stages.

1.4 Structure of the dissertation

This dissertation has a structure of six chapters, that develop the ideas proposed in this introductory chapter. The second chapter contextualizes the problem, providing information about the health sector, first in general and then particularly about the private organization under study. For the third chapter, the literature review is presented to support the study with scientific basis. In particular, it is investigated the relation between efficiency analysis and health sector, as well as the variables normally used and methodologies previously applied. The fourth chapter introduces the DEA model, and more specifically network DEA, the innovative method used. This chapter also includes the description of the models' implementation, exploring the particularities of each model. The fifth chapter unveils the case study, firstly by presenting an overview of the situation. Then, the input and output variables, data set with the respective processing for the three analyses to be performed – DEA for the Hospital, DEA for each selected service and network DEA. Chapter five will also include the obtained results and its discussion. Finally, in the chapter six, the conclusions of the dissertation are presented, along with limitations found during the proposed approach and suggestion for future prospects.

2

Context and problem

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The Health care sector is a particular one since it incorporates the universal human right to health. In the era of not only cost reduction to maximize profit but also the era of maximum use of the available resources, there is an opportunity space to research methods in this field. The problem and its context are described in this chapter, from the general to the specific. Firstly, there is a short description of both health care and private sector, and afterwards the Portuguese case, especially describing the particular hospital organization under study. Its story and evolution of efficiency analysis.

2.1 The health care sector

A sector is an area of the economy in which businesses share related products or services. However, it can also be thought as an industry or market that shares common operating characteristics.[1] The acceptance of the health care sector importance is clear. It consists of a business that provides medical services, manufactures medical equipment or drugs, and provides medical insurance. Its combination of social and economic factors can be extremely sensitive because the right to health is one of the fundamental rights of any world citizen (Nunes, 2016).

2.1.1 Worldwide perspective

In the last decades, the costs associated with the health care sector, in developed countries, have experienced sharp growth. Some of the reasons for that are the demographic shifts, increasing life expectancy of the population, complex and chronic diseases, expansion of coverage by public health service and technology improvements (Mossialos and Grand, 2019; Nunes, 2016). However, inefficient management of resources has also been pointed out as cause of health expenditure increase (Folland et al., 2012; Barros, 2012). Considering the resources limitation and the unlimited needs, increasing the health expenditure raises issues related to the sustainability of the system and equity of access to health care.

The sustainability problems, not only in the health sector but also in other areas, resulted in the United Nations Members States setting 17 Sustainable Development Goals (SDGs), in 2015, to be met by 2030. For instance, it includes a broad health goal, *"Ensure health lives and promote well-being for all at all ages"*.[2] However, enormous gaps remain between what is achievable in health sector and where global health stands today. Unfortunately, progress has been both incomplete and unevenly distributed.

Although it could be thought that the lack of quality services is only related to poor countries, high-income countries have fallen short on the quality scale as well. The United States reveals excess

[1] "Fundamental analysis: Sector Industires Analysis", 2019. Retreived from https://gateway.euro.who.int/en/indicators/hlthres_56-for-profit-privately-owned-hospitals-total/visualizations/#id=27851 [Accessed: March 26, 2020]

[2] WHO (2019). World health statistics 2019: monitoring health for the SDGs, sustainable development goals. Retrieved from https://www.un.org/en/sections/issues-depth/health/ [Accessed: March 26, 2020]

and preventable deaths due to poor-quality care in surgery centers, being 15% of all hospital costs in Organization for Economic Co-operation and Development (OECD) countries due to patient harms resulting from adverse events. The lack of coordination within and among health care providers and networks appears to be the greatest concern (National Academies of Sciences et al., 2018). Therefore, it can be seen as an opportunity space to improve the resources management, reducing costs without compromising quality. In another words, to guarantee the sustainability of the health system and the quality care of its patients.

Since the data will be provided by one of the main Portuguese' private hospital organizations, the analysis will now focus on the private sector growth. Private hospitals are owned and operated by an organization other than the state, and they can be categorized as for-profit and non-profit companies. The Portuguese health market is not innovative in the demand for private hospitals' growth. Several other countries, such as the USA, United Kingdom, Germany, and Austria, where the advantages of private providers are perceived, also face this reality (Kruse et al., 2018). Faster access to treatment, the opportunity to choose the health care provider, and the comfort of the surroundings are some of the main advantages felt by the patients (Sagan and Thomson, 2016). The number of private for-profit hospitals has jumped after 1990 and continued to grow until now, as represented by the WHO data in Figure 2.1.1.[3] Another interesting fact is represented by the WHO data,[4] Figure 2.1.2, where it is noticed an abrupt growth in the total number of beds in 1997 and in 2002. Demonstrating the private hospitals' tendency to grow their activity. The final relevant factor, for the growth of the private sector, is the continuous increase in the percentage of the population with health insurance. As observed in Figure 2.1.3 from a Mckinsey study,[5] it is expected to double by the year 2025.

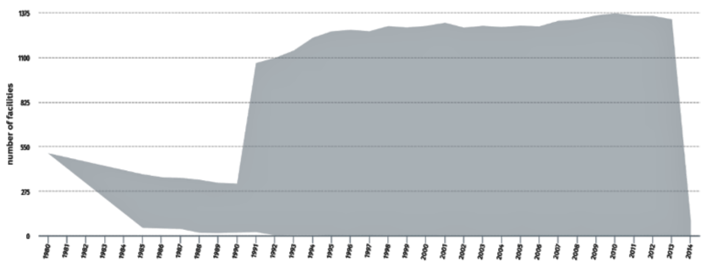

Figure 2.1.1: Total number of for-profit privately owned hospital. Source: WHO - European Health Information Gateway.

[3]WHO: European Health Information Gateway. "For-profit privately owned hospital, total number". Retreived from https://gateway.euro.who.int/en/indicators/hlthres_56-for-profit-privately-owned-hospitals-total/visualizations/#id=27851 [Accessed March 28, 2020]

[4]WHO: European Health Information Gateway. "Beds in privately owned hospital, total number". Retreived from https://gateway.euro.who.int/en/indicators/hlthres_26-beds-in-privately-owned-hospitals-total/visualizations/#id=27685 [Accessed March 28, 2020]

[5]Mckinsey global Insurance pool, from June 2016. Retreived from https://www.mckinsey.com/industries/healthcare-systems-and-services/our-insights/the-growth-opportunity-for-private-health-insurance-companies# [Accessed May 20, 2020]

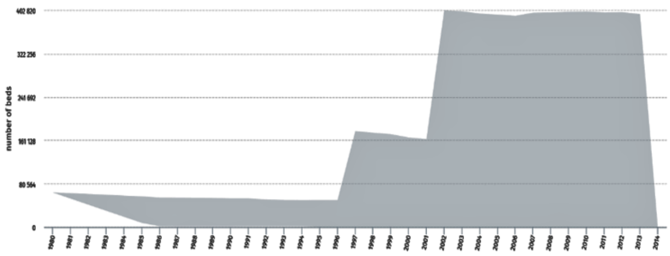

Figure 2.1.2: Total number of beds in privately owned hospital. Source: WHO - European Health Information Gateway.

A critical difference between private and public hospitals is their orientation toward financial performance. Private entities have access to the capital market and are thus incentivized to show the highest possible profit in their reports to attract investors. The importance and constant presence of methods to measure the efficiency in these organizations are influenced by this factor.

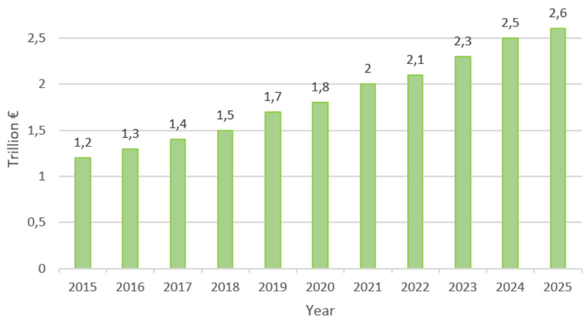

Figure 2.1.3: The expected growth of private health-insurance revenue from 2015 to 2025. Source: Mckinsey global Insurance pool, June 2016.

In countries with governments capable of using a range of regulatory and financial policy tools to control and direct mixed delivery of health services in the public interests, normally have well-established regulation of the private sector. For example, management of service access and service costs. On the opposite, in countries where the development of private-sector regulation is limited and regulatory capacity is not developed, this sector and mixed health systems does not operate consistently with the country's health goals (Brugha and Zwi, 1996). Another factor that provides relevance to the private sector is the recognition of its importance by governments.[6] Many challenges have been faced over the last two decades, including fiscal space constraints arising from financial crises, changes in disease burden (especially towards chronic, noncommunicable diseases), demographic shifts, population displacement, and political and economic instability (Brugha and Zwi, 2002). Therefore, governments

[6] United Nations (1997). Guidelines for Private Sector Participation in Ports. Retrieved from https://www.unescap.org/sites/default/files/pub_1855_fulltext.pdf. [Accessed: March 30, 2020]

perceive the private sector as a possible solution to these problems. Offering access to greater service capacity, greater responsiveness, managerial expertise, technology and innovation, and investment and funding.[7] Although there are several noticed advantages in the private sector, there is one factor which must be recognized first, the lack of published information in this area.[8] Studies where the performance is evaluated are even rarer. These challenges should be addressed with brevity since the private sector has been developing and serves more than half of the population in some countries (Basu et al., 2012).

2.1.2 The Portuguese case

It was only after Portugal became a member of the current EU, in 1986 that social infrastructures were developed through access to European funding. It made viable to access and promote equipment expansion. In 1990, the Basic Law on Health was decreed, being a decisive turning point for the National Health Service (SNS). It defined the state's role as responsible for health care, operating through the creation of its own services and agreements with private partners.[9] The privatization of health care providers was enabled. Resulting in the development of the private sector and the private management of public health care facilities.

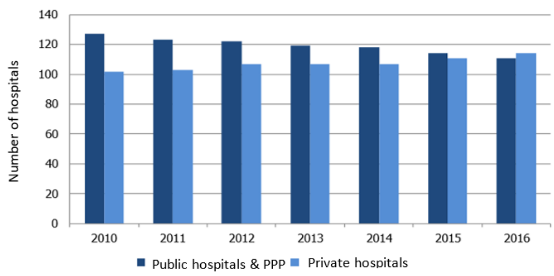

Figure 2.1.4: Number of hospitals according to institutional nature, Portugal, 2010-2016. Source: INE.

In the post-crisis, after 2011, it has been detected an abrupt growth in the participation of private hospitals in the public health. Along with an increase in the number of private hospitals, Figure 2.1.4. From the 230 Portuguese hospitals in 2018, represented in Figure 2.1.5, more than half were private hospitals – 119, 51.7% [10,11]. Between 2015 and 2016, the number of emergency services, medical consultations, complementary diagnostic acts, and complementary therapeutics acts in hospitals increased more sig-

[7]Clarke, D., Doerr, S., Hunter, M., Schmets, G., Soucat, A., and Paviza, A. (2019). The private sector, universal health coverage and primary health care. technical series at the global conference on primary health care.Retrieved from https://www.who.int/publications-detail/the-private-sector-universal-health-coverage-and-primary-health-care. [Accessed: March 30, 2020]

[8] Balabanova,D.,Oliveira-Cruz,V.,and Hanson,K. (2008). Health sector gover-nanceandimplicationsfortheprivatesector. Retrieved from https://www.r4d.org/resources/health-sector-governance-implications-private-sector/. [Accessed: Abril 31, 2020]

[9]Law-Decree (Decreto-Lei) no 48/90, of 24th August

[10]Portal do INE. "Hospitais (N.º) por Localização geográfica". Retreived from https://ine.pt/xportal/xmain?xpid=INE&xpgid=ine_indicadores&contecto=pi&indOcorrCod=0008101&selTab=tab0 [Accessed March 3, 2020]

[11]Portal do INE. "Dia Mundial da Saúde - 7 de abril". Retreived from https://www.ine.pt/xportal/xmain?xpid=INE&xpgid=ine_destaques&DESTAQUESdest_boui=313635671&DESTAQUESmodo=2&xlang=pt [Accessed March 3, 2020]

nificantly in private hospitals than in public or non-profit private (PPP) hospitals. Nonetheless, the activity is still majorly performed in public hospitals (Footnote 11). For instance, 84.6% of the emergency services of 2016 were performed in public hospitals or PPP. Nonetheless, the number of beds, which is normally used to quantify the hospital's activity, has also increased in the private sector. The same was not verified for the public sector, Figure 2.1.6. However, the increase of the activity in the private sector is not expected to stall in the following years, since in the next decade is projected to open thirteen new hospitals until the end of 2020 [12]. This growth of private hospitals is in concordance with the number of emergency services. In 10 years (2006-2016), it almost doubled from 665 thousand patients to 1.2 million. Moreover, there is also a growth in the percentage of the population which benefits from health insurance, Figure 2.1.7. In concordance, with a study performed by Marktest, Basef Seguros in 2020,[13] 3 million and 12 thousand Portuguese benefit from health insurance, which corresponds to 33.5% of the Portugal residents. This fact reinforces the notable increase of the importance of the private sector.

Figure 2.1.5: Distribution of the number of hospitals according to institutional nature, Portugal, 2017 and 2018 Source: INE.

Most of the private hospitals are part of a hospital and clinic networks, with the major companies being the José de Mello Saúde, SA (the health care division of the Mello group), Luz Saúde, SA (formerly a division of the Espírito Santo Financial Group and now part of the Fidelidade insurance group) and the Lusíadas Saúde SGPS, SA (formerly part of the United Health care Group).

2.2 Hospital efficiency

Analyze and improve efficiency methodologies is one of the main focus of this dissertation. After all, it is proposed a new methodology. Therefore, it is necessary to understand the importance of these measurements for society, and identify the connection between efficiency and the health sector. In

[12] Diário de Notícias. "Criados pelo menos 19 hospitais privados em Portugal". Retreived from https://www.dn.pt/edicao-do-dia/02-ago-2019/criados-pelo-menos-19-hospitais-privados-em-portugal-11168921.html[Accessed March 4, 2020]

[13] Marktest, Basef Seguros study, from 2020. Retreived from https://gs1pt.org/news/posse-de-seguro-de-saude-em-crescimento/ [Accessed Jun 5, 2020]

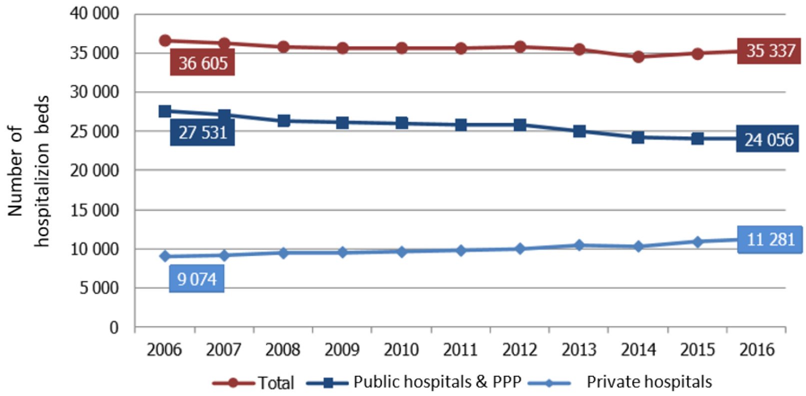

Figure 2.1.6: Hospitalization beds according to institutional nature in Portugal, 2006-2016. Source: INE

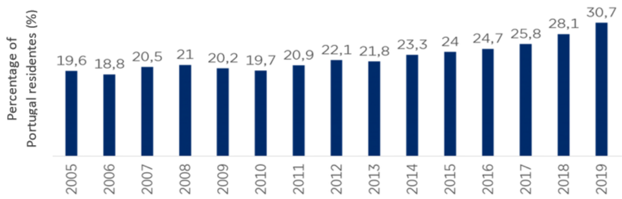

Figure 2.1.7: Percentage of Portugal residents which benefit from health insurance, from 2005 to 2019. Source: Marktest, BasefSeguros, 2020

particular, the difference between private and public efficiency approach.

The efficiency of hospital units is linked to the term organizational performance, between planning and control. While the first focuses on the whole dynamics from the identification of objectives to the strategies to achieve them; the second manage the company so that the proposed goals are reached (Atkinson et al., 2000). According to Slack et al. (2002), all organizations must measure their performance in order to obtain better results. Although, the external hospital's environment can be studied separately, the internal aspects, such as daily hospital activity, should be analyzed in detail to understand its effects on performance.

Nevertheless, despite cost rising, hospital efficiency has been an ignored political preoccupation in many countries (Färe et al., 1994; Davis et al., 2013). Thankfully efficiency analysis has been becoming a great concern for the academic, impelling an increase of interest from the political society, also concerned about the limited resources (Jacobs, 2001; Davis et al., 2013). Therefore, the maximizing efficiency and quality in hospitals has started to become an essential dynamic for hospital administrators.

Although for Gok and Sezen (2013) efficiency is achieved when inputs are consumed without waste, and the outputs are maximized; for Kirigia et al. (2008) efficiency means obtaining maximum resources or minimize the use of available resources to produce a certain level of services in the hospital context. Regardless of the efficiency definition used, health organizations, and consequently care quality, could achieve multiple enhancements if their efficiency was improved.

Most researches related to hospital performance have focused on measuring technical and costs efficiency through methods such as DEA and other statistical models. Ignoring the internal connection between services. In short, this definition of efficiency goes through the best possible combination of inputs for reversing the maximum output. However, the methodology description will be more completely done in the following chapters.

2.3 CUF

This dissertation, as previously stated will be done in collaboration with CUF. Therefore it is essential to do a brief presentation of their background. Formerly known as José de Mello Saúde, CUF is the largest private health care provider in Portugal. Founded in 1945, in Lisbon, it holds 75 years of experience in the provision of state of the art technology, medical devices, and scientific investigation, as well as, utmost comfort for customers. Its mission focuses on becoming a prime example of clinical and human capital excellence through the ongoing growth of an integrated network of hospitals and clinics – 9 private hospitals, 8 clinics, 1 institute, and 1 public hospital. Indeed, CUF's yearly numbers have been corroborate these claims, Figure 2.3.1.

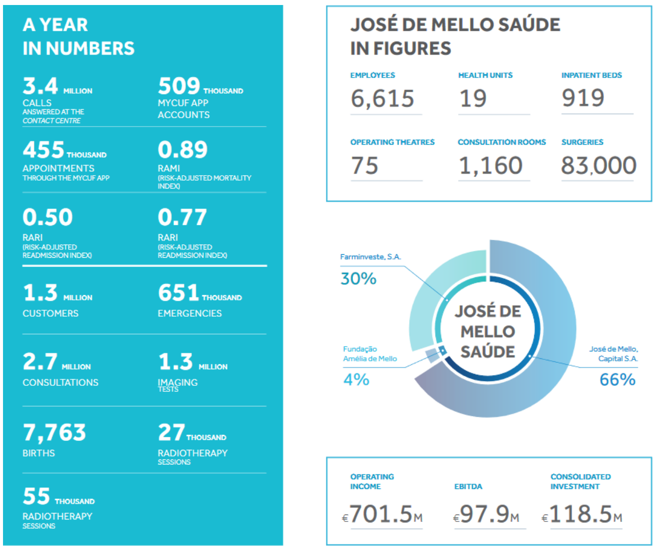

Figure 2.3.1: The numbers of José de Mello Saúde's Integrated Report 2019, Page 11

Considering the company's large dimension, one can only expect an even larger amount of data associated with daily business operations. Undoubtedly, Information management (IM) and Business Intelligence (BI) are at the forefront of strategic decisions and technological innovation; without such structures, one deprives the organization's empowerment. These tools are enablers of efficiency and productivity since they allow the firm to gather new market insights, to assess supply and demand, and to accurately evaluate the impact of business decisions (Chugh and Grandhi, 2013).

2.3.1 Data collection and analysis

CUF understood the implications of a sound data structure and the need for more, better, and faster data grew, so did the idea of investing in a new project – the Go Forward Program. CUF decided to redefine its Health Information Technology (HIT) strategy, moving away from a function-specific perspective and towards a more integrated, modular, interoperable, and agile direction for its future technology-related tools and infrastructure.

Initially, a diverse range of operating systems collected and stored data – about different departments within the organization, whether clinical or not – into separate unrelated silos that accepted disparate data formats and which could not be connected. Needless to say, responding to any analytical demand posed quite a challenge, given that certain data could be scattered across all of these systems – requests for structured and robust reports and dashboards, in addition to being inordinately time-consuming, were too complex for frequent use by end-users and inefficient at producing results promptly.

As a consequence of these limitations, the DW Project was born as an attempt to provide a much needed analytical framework, cloud-deployed. Moreove, it could also support the IM process and, concurrently, solidify a metadata repository (single and coherent variable definition), build a canonical model (applicable to all types of data, clinical and non-clinical), standardize critical business variables (to be further explored through departmental dashboards and analyses), devise sophisticated analytical models (to be integrated within daily operations, thus harnessing internal data value), and discontinue the usage of previous inefficient data sources.

To ensure all these requirements are consistently met, three main objectives were defined: Single Data Source, Data Migration from obsolete analytical systems, and Swift Response to the organization's needs – designing an evolving structure that accounts for corporate growth whilst maintaining data consistency and integrity, thereby a pivotal condition.

Concisely, the DW Project aims to provide better access to reports and data analyses so that both Management and Clinical teams will be able to anticipate, support, and validate their daily decisions through the most recent professional knowledge available – especially significant for clinicians when diagnosing and treating patients.

2.3.2 Efficiency analysis

The data acquisition previously explained is an essential first stage in order to perform efficient analysis. The electronic records are not a recent subject when it comes to the CUF routine. Accordingly, the activity and billing analysis have been facilitated. Nonetheless, the increasing number of clinics and hospitals, as well as the number of informatic applications have brought challenges.

The analysis made, in historical terms, for control and efficiency issues requires the use of differentiated human resources' knowledge and the Microsoft Excel calculation tool. In regards of efficiency language, as will be explained in Chapter 3, the efficiency at CUF is based on ratio analysis. The technique relies on the number of performance dimensions. Consequently, it is a dependent methodology.

Although an investment path has been made in CUF, especially regarding the use of business intelligence tools, what is proposed in this thesis is an innovative approach - network DEA - that correlates the different hospital's dimensions. However, to evaluate the results obtained in this disruptive methodology, the DEA analysis, which is the most common efficiency methodology, will be firstly performed (Hollingsworth, 2016; Kao, 2017).

2.4 Summary

This chapter served to present the context where this dissertation is based on. First by characterizing the health sector and then entering the private domain, worldwide and in Portugal. The previous sections include the explanation of the importance of future goals in health around the world, the evolution of the private sector in health, and the evolution of efficiency analysis. Lastly, an overview of the organization under study is accomplished. In this subsection, the organization's history is introduced, as well as the most recent investments in tools for data acquisition. The process of writing this section allowed the understanding of how their system works, the importance given to efficiency analysis, and the gap in the methodology used. Finally, making use of this gap, this thesis concludes the identification of its opportunity space.

3

Literature review

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3.2 Efficiency model elements 27
3.3 Efficiency measurement using DEA and its adaptions 33
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Efficiency is the ability to avoid wasting materials, energy, efforts, money, and time in producing a desired result (or output). However, the term is used differently in many sectors depending on the pretended inputs and outputs, resulting in multiple analysis' methodologies.

This dissertation presents an innovative performance measurement model applied to a case study in healthcare sector. Therefore, the objective is to perform a literature review on previous studies in health efficiency, facilitate the definition of relevant models, inputs and outputs variables to analyse efficiency. So that later it will be possible to validate the utility of the applied model. In the section 3.1, general concepts on performance measurement in healthcare are provide, as well as typically employed methods. Section 3.2 provides a guide throughout the fragments that assemble the efficiency models. For the last section of this chapter, efficiency measurements using DEA and its upgrades are addressed, focusing on its theories and components, and finalising with the knowledge gap that justifies the creation of the proposed model.

3.1 Measuring performance in health care

In the first section of this chapter, general concepts on performance measurement are provided to facilitate the comprehension of the next sections. The section starts with the definition of concepts and then provides the most used method when measuring efficiency. Besides that, it will also be emphasize the importance of measuring efficiency in healthcare sector.

3.1.1 Technical and allocative efficiency

Farrell (1957), one of the biggest economists in the twentieth-century, defined efficiency as "*the firm's success to produce the maximum feasible amount of output from a given amount of input or producing a given amount of output using the minimum level of inputs where both the inputs and the outputs are correctly measured*". Three different types of efficiency were also introduced by Farrell: technical efficiency, allocative efficiency, and economic efficiency.

Definition 3.1 (Directorate-General for Health and Food Safety (European Commission), 2019)[1]. Technical efficiency (TE) indicates the ratio between the useful work performed by a system and the total energy consumed as input. In another words, it indicates the extent to which a provider is securing the minimum cost for the maximum quality in delivering its agreed outputs, independent from the value placed in these outputs.

Definition 3.2 (Ravaghi et al., 2019; Directorate-General for Health and Food Safety (European Commission), 2019). Allocative efficiency (AE) addresses the issue deploying the resources towards prod-

[1] Smith, P. C. (2009). Measuring for value for money in health care: concepts and tools. Technical report,Centre for Health Economics: The Health Foundation. Retrived from `http://www.health.org.uk/sites/health/files/MeasuringValueForMoneyInHealthcareConceptsAndTools.pdf`

ucts or services that maximizes the welfare according to social values, or alternatively, if the costs of the chosen inputs are the minimum feasible.

Definition 3.3 (Farrell, 1957) Economic efficiency is AE and TE from a joint unit of cost efficiency.

In the healthcare sector, efficiency is one of the most discussed dimensions of healthcare performance (Cylus et al., 2016b). The improvement of resource management is essential, not only because resources are limited, but also because WHO defined the world objective of achieving Universal health coverage (UHC). In order to pursue this goal an increase on money value is necessary. Nevertheless, it is crucial to analyse its current value and for that several studies have been published in health performance analysis.

The importance of collecting data is not as recent as perceived. For instance, there is evidence that patient outcome data were being collected at the hospital of the University of Pennsylvania as early as the middle of the 18th century (Colton, 2000; McIntyre et al., 2001). To analyse the available data, there are two major approaches, parametric and non-parametric. While parametric statistics assume that data follow a probability distribution based on a set of fixed parameters. In other words, a functional form linking inputs to outputs. Non-parametric statistics is not based on parameterized families of probability distribution. Actually, the parameter set can change when new information is introduced to statistical tests, providing more flexibility but less robustness. Nonetheless, both approaches have been applied, in the last decades, to performance measurement and analysis of healthcare, as presented in literature table A .

Maniadakis and Thanassoulis (2004) have used input prices to develop a productivity index, non-parametric, inspired on Malmquist Index (Malmquist, 1953). By the year of 2004, allocative efficiency has been rarely incorporated in productivity measurements. So, this study introduced a decomposition of productivity change that captures changes in TE and AE, programmed using DEA, to better determine the root sources of changes and provide valuable information for decision making. However, input prices are required to apply the developed method. Other studies have also analysed AE, Herr et al. (2011), for instance, investigated cost and profit efficiency in German hospitals and their variation with ownership type. Previous studies were not concordant on which type of organization were effectively more efficient, and so privatization keep increasing in Germany. Noticing that profit efficiency could be manipulated by extending the lengths of stay, Herr et al. (2011) used SFA, parametric method, on a multifaceted administrative German dataset, which included balance sheets of 541 hospitals of the year 2002-2006. In order to analyse efficiency in both organizations, they compared cost and profit efficiency, since the maximization of profit can be obtained at the expense of higher costs. Both AE and TE are included in the profit efficiency analysis in order to reduce data manipulation. The results obtained indicated that private hospitals were not significantly less cost or less technical efficient than public institutions. However, private ownership was associated with higher profit efficiency compared with public ownership in the time period from 2002 to 2006. Nevertheless, this study was aware of the lack of information about healthcare quality, which was analysed by Nayar and Ozcan (2008) and Hu et al. (2012). Using a sample of Virginia hospitals, Nayar and Ozcan (2008) performed technical efficiency analysis while examining quality performance. Previously to this study, DEA has been usefully used to assess technical and allocative efficiency, but little had been done to include quality measure. After applying this methodology, they found that technically efficient hospitals were performing well as far as quality measures were concerned, but also that some technically inefficient hospitals were performing well with respect to quality. Similar conclusions were found by Hu et al. (2012) when investigating regional hospital efficiency in China during the 2002-2008 period especially regarding on a new health insurance reform. They chose to also apply DEA, since it is a methodology that better handles multiple outputs. For instance, the undesirable output - patient's mortality - to evaluate the quality of care delivery. After analysing the obtained results, it was perceived a slightly increased of hospital efficiency from 0.6777 to 0.8098 during the sample period. However, this was not the only retreated conclusion, since the regression analysis suggests that higher proportion of for-profit hospital and high-quality hospital is helpful to enhance TE; while government subsidy has a negative impact on coastal regions' hospital efficiency. Crucially, they found that the new health insurance reform had a significant efficiency-enhancing effect.

Nonetheless, analysis of TE is still more common, since it does not require a previous specification of the norms, and instead data can be entirely examined a posteriori. It doesn't mean, however, that the quality of health delivered cannot be taken into account. Campanella et al. (2016) investigated technical efficiency of Italian hospital care. In order to include the quality analysis, 30-day risk-adjusted of mortality, as well as diseases diagnosis, were used as output variables. There was found, through the application of the DEA methodology, a positive association between efficiency and a lower case-mix index. In the north of Italy, region with fiscal autonomy, higher public and a lower private expenditure on health as percentage of Gross domestic produc (GDP) is observed.

3.1.2 Efficiency evaluation methods

Several methodologies, parametric and non-parametric can be used to evaluate hospital performance. From the literature review and in agreement with Ozcan (2008), viz.: ratio analysis, Least-Squares Regression (LSR), SFA, and DEA.

3.1.2.1 Ratio analysis

Mainly used for efficiency (productivity) in healthcare financial management, ratios are the relationship and comparison between one input and one output (Martins, 2004). One of the simplest and oldest performance calculation methods found in literature, and still used in some companies.

$$Efficiency(Productivity) = \frac{Output}{Input} \tag{3.1}$$

Ratio analysis is dependent on the number of performance dimensions, which have to be aggregated into compatible units or within one unit over different time periods. Therefore, several ratios have to be computed.

Caballer-Tarazona et al. (2010) investigated the efficiency of three healthcare service units of 22 hospital in the Valencian Community (East Spain). In their study, DEA was used along with two efficiency indexes $\frac{Admissions_{Weighted}}{Doctors}$ and $\frac{Interventions}{Doctors}$. After performing a discriminant analyses, the effectiveness of these two indexes was proved. Resulting in a methodology for measuring efficiency easier and as effective as DEA model. However, it is necessary to take in account that it was applied to one service and not to the complex organization, that is an hospital. Caballer-Tarazona et al. (2010) also referred this aspect by emphasizing the accuracy of obtained information when services are study instead of the general hospital efficiency.

Nonetheless, services do not always perform separately from the hospital, resulting in vast amounts of ratios that frequently produce mixed results which complicate the decision-making process of health managers in comparative performance analysis. Taking that into consideration, healthcare managers have difficulties identifying a consistent benchmark that incorporates all inputs and outputs of a healthcare organisation (Ozcan, 2008).

3.1.2.2 Least-Squares Regression (LSR)

By allowing multiple inputs and outputs, as well as considering the noise, LSR – also known as Ordinary Least Squares (OLS) - is one of the most popular parametric methods in efficiency evaluation. Generally, it is formulated as:

$$y = \beta_o + \left(\sum_{i=1}^{n} \beta_1 x_1 \right) + e \tag{3.2}$$

where y has a normal distribution for any fixed value of x (being they value independent of one another), the mean value of y is a straight-line function of x that includes the error term, e. The variance of y is the same for any x, and, for any fixed value of x, y is a random variable $(y \mid x) = \beta_0 + \beta_1 x$.

SFA using both cross-sectional and panel data - where assumptions are relaxed -, as well as OLS analysis were applied by Mateus et al. (2015). The efficiency comparison was done in four countries – England, Portugal, Spain and Slovenia. As result, when applying likelihood ratio test, cross-sectional data SFA is not statistically different from OLS in Portuguese data, while SFA and OLS estimates are statistically different for Spanish, Slovenian and English data. Concluding that, panel data are preferred over cross-section analysis because results are more robust, avoiding heteroscedasticity. For all countries except Slovenia, beds and employees are relevant inputs for the production process.

Benefits in LSR are recognised when technical change of a time-series data is required and when investigating scale economies. However, the LSR does not identify specific inefficient units nor the best performances, and it requires a pre-specified production function due to its parametric formulation (Ozcan, 2008).

3.1.2.3 Stochastic Frontier Analysis (SFA)

Another, already mentioned parametric method is SFA. This technique is used to estimate production and cost functions, while explicitly accounting for the existence of inefficiency. When comparing to other parametric method, SFA assumes that deviations from the efficient frontier are due to a noise factor. A generic SFA model is normally formulated as:

$$\textit{Total Cost (TC)} = TC(Y, W) + V + W \tag{3.3}$$

where Y represents outputs, W connotes input prices, V depicts the random error (assumed to be normally distributed with zero mean and zero variance), and U acts as the inefficiency residual.

In the literature review Carey (2003), Herr (2008), and Herr et al. (2011), decided to use SFA as the methodology to analyse hospital efficiency.Carey (2003) performed stochastic frontier cost function to investigate the economic theory that suggests that consolidation of hospital ownership through formation of multiple hospital “systems” improved hospital performance. This paper uses data that is differentiated among systems according to their strategic and structural features. The study concludes that organization of physician and insurance activities’ efficiency improves at the system level. Herr (2008) and Herr et al. (2011) were more focused on understanding the effects of ownership in efficiency. While in 2008 technical and cost efficiency were analysed; in 2011 profit efficiency analysis is also performed. The first study concluded that private hospitals were less cost efficient than publicly owned hospitals, while the second notice an insignificant difference. The different in results is justified by change in the remuneration system for German hospitals. Regardless of that, private hospital presented a higher profit efficiency but negative technical efficiency, since highest length of stay – common in private hospitals – is negatively correlated with efficiency.

Taking all of this into consideration, SFA is useful for hypotheses’ testing and to measure not only technical and allocative efficiencies, but also profit and cost analysis. However, results are, frequently, highly sensitive to the estimation decisions made. The challenges associated with model construction, such as specification of functional form, identification, and extraction of efficiency estimates promoted the developed of other methods, such as DEA (Jacobs et al., 2006).

3.1.2.4 Data Envelopment Analysis (DEA)

The first, and only, non-parametric technique explained in this thesis is DEA. In the last three decades, DEA has been used in more than 90% of the efficiency analysis in the health sector (Hollingsworth, 2016; Kao, 2017).

In opposition to SFA, DEA assumes that not all units are efficient and allows the use of multiple inputs and outputs in a linear programming model. At the end, it computes a single efficiency score per observation. DEA, and most recent variations, will be described in the Subsection 3.3.

Nonetheless, DEA also presents limitations. For instance, since it is deterministic it does not incorporate noise as SFA. Therefore, DEA is more susceptible to the outliers; otherwise units would become inefficient due to deviations from the efficient frontier (Hollingsworth, 2003; Ruggiero, 2007).

3.1.3 Public and private sector

Regarding the differences between public and private sector applied techniques, no differences have been noticed. The most common methods are, for both sectors, DEA and SFA when it comes to measuring health efficiency (Hollingsworth, 2003, 2008; Hollingsworth and Scott, 2012; Hollingsworth, 2016; Jaafaripooyan et al., 2017).

Several papers have actually compared the efficiency of private (for-profit and non-profit) and public hospitals, therefore using the same methodology for both sectors. For instance, Marinho (2001), Tiemann and Schreyögg (2009), Tiemann and Schreyögg (2012), and Hsiao et al. (2018). Performance index were frequently used in Brazil, so Marinho (2001) decided to complement this approach with DEA to evaluate efficiency. At the end of the study, he concluded that DEA provided more consistent results. In Germany, Tiemann and Schreyögg (2009) evaluated the efficiency using a bootstrapped DEA, obtaining that public hospitals perform significantly better than private ones. They also found a positive relation between hospital size and efficiency, and a negative impact of competitive pressure. As a conclusion, their results suggested that private for-profit hospitals place greater emphasis on earning profits (*i.e.*, higher revenues per case due to higher prices), whereas public hospitals, because of resource constraints, focus primarily on input efficiency. However, Tiemann and Schreyögg (2012) discovered that conversion from public to private for-profit status promoted an increase in efficiency, that appear to be permanent. The increase was mainly achieved through decrease in all staffing ratios, with exception of physicians and administrative staff. In 2018, (Hsiao et al., 2018) also performed DEA to analyse Taiwan hospitals efficiency. Their results showed that private hospitals were more technical efficient than public hospitals, while, in terms of scale efficiency public hospitals have higher efficiency.

3.2 Efficiency model elements

In the section 3.2 performance measurement models are segmented into their different components to gather a more complete batch of information.

3.2.1 Efficiency measures

At the beginning of this chapter, efficiency was defined by Farrell (1957). Through his definition, efficiency optimization results from maximizing the outputs or minimizing the inputs, improving the output/input ratio, as represented in the Figure 3.2.1.

Figure 3.2.1: The naive view of efficiency. (Source: Cylus et al. (2016a))

However, several issues can arise when seeking a simplistic model, because it is uncapable of reflecting the complexity of the healthcare production process. According to Cylus et al. (2016a), five aspects of any efficiency indicator must be assessed: (1) the organization to be analysed; (2) the outputs; (3) the inputs under consideration; (4) the external influences to the realization; and (5) the connection with the remaining health system.

3.2.2 Efficiency model components

These five aspects presented by Cylus et al. (2016a) suggest that the simple notion of efficiency, as the conversion of inputs into valued outputs ignores a series of conceptual and methodological factors. In order to overcome the simplistic notion, Smith (2009) illustrated the efficiency problem as represented in Figure 3.2.2 (Footnote 1). The system inputs should also incorporate previous investments as represented by the box with endowments of the previous year and external constrains; while the system outputs should include endowments for the next year, joint outputs and the ones directly related to health, such as productivity. The surroundings influence is then included in the study of health system.

Typically, healthcare efficiency models integrate multiple inputs and/or outputs. Accordingly, each variable will have relative weights, v_m and u_s, associated to each input m and output s, transcribing the relative importance of an additional unit of input or output. For a hypothetical healthcare organization

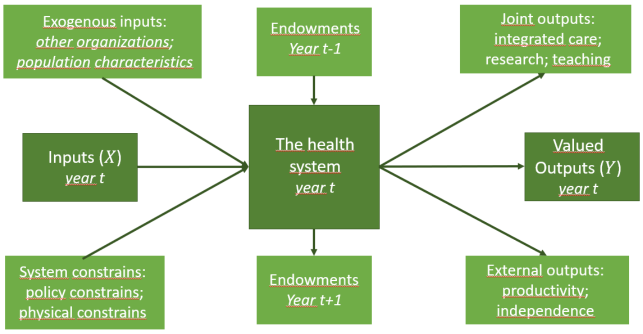

Figure 3.2.2: A more realistic development of the naïve view of efficiency. (Source: Smith (2009))

0, it is possible to calculate the valuation of inputs, $X = \sum_{m=1}^{M} v_m X_{m0}$; and the valuation of outputs, $Y = \sum_{s=1}^{S} u_s Y_{s0}$. Note that in competitive markets, both, v and u, might be already available. These fixed weights approach makes the efficiency calculations trivial. However, in complex contexts such as healthcare, weights are rarely known, particularly on the output side, and that is where analytic techniques such as DEA emerge providing total flexibility in weights (Jacobs et al., 2006).

3.2.2.1 Unit under scrutiny

As explained by Cylus et al. (2016a), when performing a efficiency analysis a clear idea of the entity under investigation should be obtained. However, it is also important to acknowledge that its performance will be influenced by other health entities or by factors beyond control. According to, Jacobs et al. (2006), three criteria should guide the choice of units:

i. They should capture the entire production process of interest;

ii. They should be DMUs, i.e, their function should be to convert inputs to outputs and to be discrete about the technological conversion process that takes place;

iii. They should be comparable especially in the sense that they are seeking to produce the same set of outputs.

Definition 3.4 (Kao, 2017) Decision making unit (DMU) is an organization under the investigation in the light of DEA. Profit-pursuing, government, and non-profit hospitals are examples of facilities whose performance is adequate for DEA.

In the literature, dozens of articles are found using different DMUs while applying DEA. See, for instance, Gruga and Nath (2001), Sahin et al. (2011), Sommersguter-Reichmann and Stepan (2015), and Stefko et al. (2018). While the first investigated the impact of ownership, size, and location on the relative TE of community hospitals in Ontario; the second adopted the Malmquist index to analyse the operational performance of the 352 Ministry of Health's general hospitals, in Turkey, during the period

2005–2008. The focused can also be directed to analyse the performance of assessment of in patients in Austrian hospital, as done by Sommersguter-Reichmann and Stepan (2015) or to the disparities and discrepancies between hospitals from different regions (Stefko et al., 2018). From this reduced sample of studies, one can notice the variety of DMUs where DEA is applied.

However, one major aspect is the definition of the DMU boundaries, since it could be thought as the entire health system. Defining the system and identifying its fundamental decision makers and inputs is not entirely clear. Notice that taking interest in single actions is preferable in comparison to a whole-system approach, since it facilitates the identification of inputs and the activities are limited. Nonetheless, outputs are generally produced by team and entities that might be linked to each other, making it hard to separate single actions. So, the most appropriated approach for analytical evaluations is a large collection of individuals (Jacobs et al., 2006).

3.2.2.2 Health care inputs

Inputs are generally easier to measure than outputs, especially because they can be aggregated into one single financial input as costs (Jacobs et al., 2006; Cylus et al., 2016a).

Definition 3.5 (Cylus et al., 2016a) Inputs consist in any resources that are used with the goal of producing health care outputs and/or outcomes, both monetary or physical resources as well as health care activities (diagnosis tests and surgical procedures) can be considered.

However, for a long-term perspective, the level of disaggregation of inputs to be specified is dispensable. The use of a single measure of costs basically transforms the efficiency model into a cost function, which will directly reflect in cost efficiency. As for the short-term analysis, the inputs may need to be disaggregated to capture the different input mixes of the organization (Jacobs et al., 2006). The two types of inputs are:

1. **Labor Inputs**

   Labour inputs can be measured by skill level of disaggregation, or, when there is no specific interest in the deployment of different labour types, aggregated into a single measure, by associating their specific weights according to their wages. Aggregation allows the analysis to focus on aspects of the production process where there is less evidence on weightings.

   This type of inputs can be measured in either physical units (hours of labour) or costs of labour, and their use depends on the context where the efficiency model is built. While the use of physical inputs will fail to capture variation in wage rates, it may desirable if the variation results from factor beyond the control of the organization.

   However, the use of labour inputs becomes increasingly difficult when the unit of observation within the hospital becomes smaller. For instance, staff normally works across sub-units, but information or financial systems cannot track their input across these units.

2. **Capital Inputs**

   Capital inputs are not less challenging, because of the difficulty of measuring capital stock and the issues in attributing it to a particular time. Often measure of capital are vulgar and ambiguous (e.g., the number of hospital beds as a proxy for physical capital). Moreover, healthcare organization frequently invest in non-physical capital inputs.

   Theoretically, an efficiency model uses the capital consumed in the current time period as an input to production process. However, the outputs of one specific year, t, may rely on investment from previous years, $t-1$, or current investments may be expected to produce later results, $t+1$, as expressed in the Figure 3.2.3. Then the output/input ratio for that time period, t, will not provide reliable efficiency measurement.

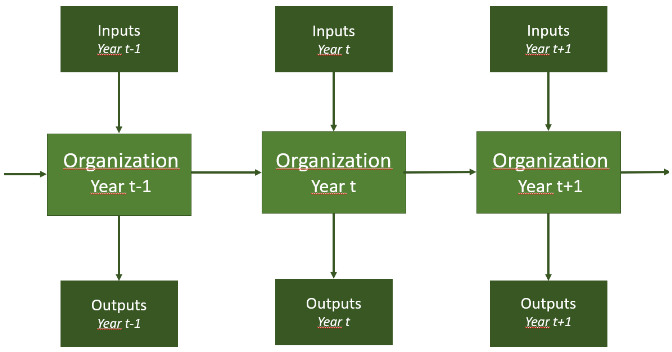

Figure 3.2.3: The dynamic nature of organisational performance. (Source: (Jacobs et al., 2006))

   This aspect results in different interpretation, depending on the analysis being short- or long-termed, which should be taken into consideration to avoid imprecise evaluations. While, for short-term analysis, the hospital available capital must be meditated; for long-term analysis, improvements made through capital investment should be included in the analysis.

Based on the literature review of thirty articles (see table A), for each study it was identified the system' variables, inputs and outputs, and the model used. As for the inputs used, the quantification of the variables is represented in Figure 3.2.4. by the number of studies which used them.

Note that not all the variables presented in the Literature Table A are included in the graphs, since some of them were not relevant, due to the low use or excess of specification. It is also important to refer that some of the studies used different denominations for the same variable or specific branches. For instance, in the number of staff, while some studies specified different specialization, other did not. As for the quantification presented, all of them were considered as the general number of staff, if more than two specializations were defined in the study under consideration.

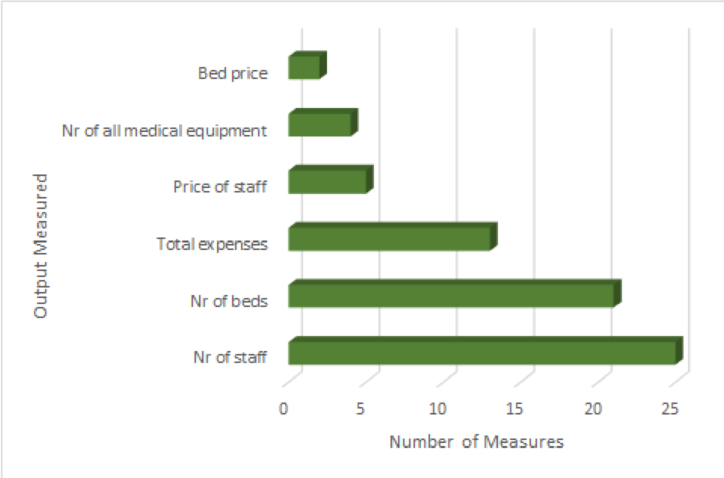

Figure 3.2.4: Inputs used in Hospital efficiency measures abstracted from published literature

3.2.2.3 Health care outputs

Defining outputs of health care sector is challenging because it is rarely demanded for its own benefit. Rather, the system's outputs are viewed in terms of contributing to health gains. Therefore, the health care outputs should be properly defined as health outcomes produces. Taking that into account, it is possible to distinguish two types of outputs (Hussey et al., 2009; Cylus et al., 2016a):

- Health services, which consists for e.g in visits, drugs and admissions;
- Health outcomes, for instance, preventable deaths, functional status and clinical outcomes.

Nonetheless, information about health outcomes is rarely collected. In addition, even when it is collected, the outcomes are hard to compare, since clinical context widely vary in terms of patients, conditions, provider and geographic area (Schuster et al., 1998; Fiscella et al., 2000). Taking into account these quality differences, adjustments need to be made to overcome this difficulty (Hussey et al., 2009).

The application of Patient-Reported Outcome Measures demonstrates progresses in assuring secure comparisons, in specific for providers delivering a specific treatment (Weldring and Smith, 2013). However, it is still more common that analysts rely on the quantity and type of activities the organization undertake. In truth, relying on this type of outputs, considering research evidences, gives rise to health improvement, and measuring such activities provides a powerful indicator of expected health outcomes. At last, the employment of activity measures is recurrently the sole option at hand, but one must be aware of its limitations (Jacobs et al., 2006). As done for the input, it was also quantified the outputs used in other efficiency' studies, represented in Figure 3.2.5.

The most common outputs were number of inpatients (22 out of 30) and number of outpatients (17 out of 30). Nevertheless, there is also studies which took into consideration outcomes, such as mortality rate and principal diagnosis.

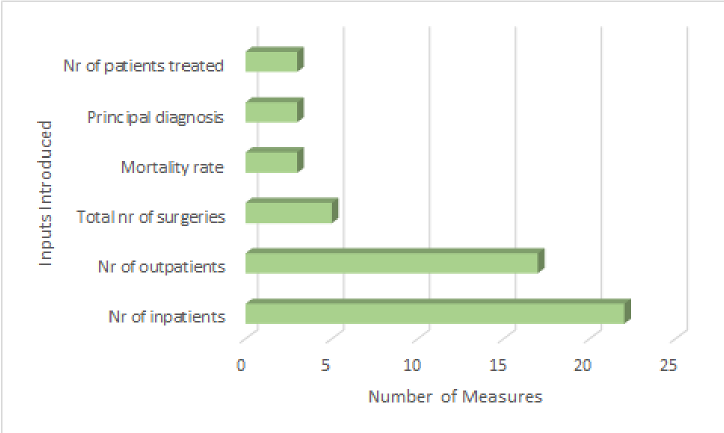

Figure 3.2.5: Outputs used in Hospital efficiency measures abstracted from published literature

3.2.2.4 Environmental influences

Every organization is affected by the external environment within which it must operate, being often greatly dependent on population characteristics. For instance, population mortality rates, surgical outcomes, and emergency ambulatory services performance heavily build upon demography, community, or geography (Jacobs et al., 2006).

In order to control such determinants, there have been several debates on the topic. For example, the performance of several healthcare organisations relies upon inputs from external agencies, and this dependency should be recognised in efficiency modelling. Jacobs et al. (2006) propose three wide-ranging manners of taking environmental constraints into account in efficiency analysis:

- Restrict the comparison to organisations with a similarly constrained environment;
- Model the constraints as being analogous to production process' factors;
- Undertake risk adjustment.

Therefore, the first approach is to accommodate comparable environmental influences. Organizations can be clustered into similar groups of similar environmental characteristics analysis (Everitt et al., 2011). However, unless exogenous influences are known to be completely independent of the efficiency, it is impossible to conclude if the variations are reflecting exogenous influences or efficiency variations. Other problem that arises is the reduction of the sample size, as an extrapolation of performance is ruled out.

The second approach is related with the incorporation of environmental factors as inputs, directly into the production modal. It effectively generalises the clustering approach solving the extrapolation problem.

The final method is the risk-adjustment. It consists in adjusting the outputs for different circumstances before they are set up in an efficiency model. In particularly, it allows the analyst to fine-tune each output for only those factors that apply specifically to that output, rather than to use environmental factors as general adjustment for all outputs (Thomas, 2004).

Regretfully, environmental constraints were not specially studied in this dissertation, but would have certainly been an even more formidable addition.

3.3 Efficiency measurement using DEA and its adaptions

The last section of Chapter 3 is focused describing DEA methodology, its usage in the healthcare context, and the specific adaptions of interest for the present dissertation.

3.3.1 DEA methodology

Nunamaker (1983) is the first published work DEA to healthcare, whereas Sherman (1984) was the first author to use DEA to evaluate overall hospital efficiency. By now there is a very extensive literature surveyed by (Hollingsworth, 2003; O'Neill et al., 2008; Ozcan, 2008; Kao, 2014).

Essentially, DEA became, in the last three decades, the predominant non-parametric method for measuring the efficiency of DMUs that use multiple inputs to produce multiple outputs. In opposition to the parametric approaches, data determines the location and the shape of the efficiency frontier. It is defined by observing of the highest output/input ratios and connects those observations up in the input-output space. Inefficient units are under the efficient frontier, being their inefficiency calculated as the difference relatively to the surface (Grosskopf and Valdmanis, 1987; Färe et al., 1994; Cooper et al., 2000). In this subsection, the technique's formulation will be addressed, the limitations and some studies which chose this approach as methodology.

3.3.1.1 DEA formulation

According to Jacobs et al. (2006), DEA assesses efficiency in two stages:

1. Identify a frontier based on either those organisations using the lowest input mix to produce their outputs or those organisations achieving the highest output mix given their inputs;

2. Create a piecewise linear envelope of surfaces in multidimensional space by assigning an efficiency score to each organisation, resultant of comparing their respective output/input ratio to that of efficient organisations.

If there are M inputs and S outputs, the production frontier becomes a surface in $(M+S)$ dimensional space. The distance of a DMU from this surface is its efficiency, which is defined as the ratio of the weighted sum of outputs of DMU divided by the weighted sum of its inputs. So, technical efficiency is computed by solving, for each DMU, the mathematical programme:

$$\max \frac{\sum_{s=1}^{S} u_s y_{s0}}{\sum_{m=1}^{M} v_m x_{m0}} \tag{3.4}$$

$$\textit{s.t} \quad \frac{\sum_{s=1}^{S} u_s y_{si}}{\sum_{m=1}^{M} v_m x_{mi}} \leqslant 1, \qquad i = 1, ..., I \tag{3.5}$$

$$u_s, v_m \geqslant 0 \tag{3.6}$$

where y_{s0} is the quantity of output s for DMU_0, u_s is the weight attached to output s, x_{m0} is the quantity of input m for DMU_0, and v_m is the weight attached to output m.

The mathematical programme looks up for DMU_0, the set of output weights u_s and input weights v_m that maximises the efficiency of DMU_0, subject to the constraint that no DMU can have an efficiency greater than 1. While the weights can take any non-negative value, and normally, are a different set of weights for each DMU. Being chosen to capture the higher level of efficiency of a certain DMU.

Equation (3.4) can be converted into a system of linear equations, erected in a way that a linear objective function can be maximised while subjected to a set of linear constraints. Nevertheless, this ratio formulation results in an infinite number of solutions (Coelli et al., 1998). Therefore, arises the additional constraint stating that either the numerator or the denominator of the efficiency ratio has to be equal to 1. Then, the problem becomes or (1) to maximise the weighted output subject to weighted input being equal to 1 or (2) to minimise weighted input subject to weighted output being equal to 1. These constrains, for option 1, can be formulated as:

$$\max_{\mu,\upsilon} \quad (\mu' \upsilon_0) \tag{3.7}$$

$$\textit{s.t} \quad \upsilon' x_0 = 1 \tag{3.8}$$

$$\mu' y_i - \upsilon' x_i \leq 0, \qquad i = 1, ..., I \quad \cup \quad \mu, \upsilon \geq 0 \tag{3.9}$$

where μ' and υ' are output and input weight vectors, respectively. This could be also expressed as the correspondent minimisation problem, option 2, (Coelli et al., 1998):

$$\min_{\lambda,\theta} \quad \theta_0 \tag{3.10}$$

$$\textit{s.t} \quad -y_i + Y\lambda \geq 0 \tag{3.11}$$

$$\theta x_i - X\lambda \geq 0 \tag{3.12}$$

$$\lambda \geq 0 \tag{3.13}$$

where x_i and y_i are column vectors of inputs and outputs for each of the $I - DMUs$, X and Y are input and output matrices representing the data for all the $I - DMUs$, θ is a scalar and λ is a $I \times 1 vector$ of constants. The value of θ obtained will be the efficiency score for DMU_0 and satisfies $\theta \leq 1$, with a value of 1 indicating a point on the frontier and hence a technically efficient DMU. The linear program must be solved separately for each DMU in the data set in order to obtain a value of θ for each DMU (Coelli et al., 1998).The objective is, therefore, to seek the minimum θ that reduces the input vector x_i to θx_i while assuring at least the output level y_i. Further considerations on returns-to-scale and model orientation will be addressed briefly in Subsection 3.3.1.2 and 3.3.1.3.

3.3.1.2 Returns-to-scale

Formally, all the previous formulations were conceptualized under Constant Returns to Scale (CRS), as postulated in the original DEA paper (Charnes et al., 1978). Six year later, Banker et al. (1984) extended this approach to a more flexible Variable Returns to Scale (VRS), appropriated when not all DMUs are operating at an optimal scale. Since, in healthcare operating at a suboptimal scale is common due to external factors, the choice of CRS or VRS is an important decision.

When adding a single convexity constrain to Equation (3.7), the CRS linear programming is converted into a VRS one.

$$\sum_{i=1}^{I} \lambda_i = 1 \tag{3.14}$$

In order to collect evidence of whether a DMU is operating in the area of increasing or decreasing returns-to-scale, a Non-Increasing Returns-to-Scale (NIRS) constraint can be added by modifying Equation (3.14) to:

$$\sum_{i=1}^{I} \lambda_i \leq 1 \tag{3.15}$$

By comparing the DMUs, TE score over the NIRS constraint to their TE score under the NIRS constraint, scale inefficiencies can then be calculated. As reported by Coelli et al. (1998), if they are not equal, then increasing returns-to-scale exist; if they are equal, then decreasing returns-to-scale apply.

At the end, the choice between CRS or VRS depends on the context and purpose of the analysis or whether short-run, productivity-based, or long-run, managerial-based (Jacobs et al., 2006). If a productivity-based perspective, it may be more appropriately to use a CRS model; if a managerial-based perspective, it is prefered to apply a VRS model.

Multiple studies on healthcare efficiency topic, perform both models, CRS and VRS, such as Grosskopf et al. (2001), Linna et al. (2006), Ng (2011), and Bilsel and Davutyan (2014). Mainly they performed both models to compared the obtained scale efficiency scores. After all, they were concordant in the obtained results. There was a slightly lower score for CRS than for VRS. Other comments on their differences were the constant slope of CRS models and a segmented display varying non-negative slopes positioned through the hospital with the highest output-input ratios for the VRS. Bilsel and Davutyan (2014) also observed that using the VRS constraint tended to increase the efficiency of larger hospitals.

3.3.1.3 Model orientation

As mention in Subsection 3.1.2.1, efficiency in ratio analysis, where focused in input reduction is done to improve efficiency. In the DEA context, this approach is named input orientation and assumes healthcare managers are focused on minimizing the inputs. However, outputs can be target of health managers, who desired to maximize them.

This difference is important to stress, since VRS results are dependent of the orientation, in opposition to CRS results, which are the same for input- and output-oriented models. Although it is incorrect to think that the choice of orientation affects which observations are identified as full efficient; the distance of an inefficient DMU is measured as horizontal for input-oriented models and as vertical for output-oriented ones (Jacobs et al., 2006).

Nonetheless, to facilitated managers decisions, some models were created combining both input reduction and output increment. It is reached by decreasing input slacks and increasing output slacks. In DEA literature, such models are designated as ADD models or non-oriented models (Ozcan, 2008). By any means, Barnum et al. (2011) found that ADD greatly overestimated mean efficiency.

3.3.1.4 DEA limitations

The non-parametric approach, DEA, allows an empirical efficiency measurement rather than a theoretical standpoint. Its flexibility of function form is seemed an attractive feature. However, some limitation can arise (Jacobs et al., 2006).

First, the location of the DEA frontier is sensitive to observations that may have unusual types, levels or combinations of inputs and outputs. Note that, DEA assumes correct model specification and that all data are observed without error.

Second, DEA uses a selective amount of data to estimate individual efficiency scores. It only compares that specific DMUs to DMUs that produce comparable mix of outputs. Therefore, if there is a unique output it will be automatically associated to full efficiency.

Nevertheless, DEA still presents as the most adaptable technique to the reality. The limitations presented are expected to be overcome, by comparing the same organisation over several periods of one month, in the last three years (12 months per 3 years = 36 comparable periods).

3.3.2 Network DEA

In contrast to conventional DEA, where a system is considered as a "black-box", network DEA acknowledges its internal assembly to generate more enlightening results. Some possibilities, that are not considered by the common DEA are: (1) that a system may be globally efficient, even if all its subsystems are not (Kao and Hwang, 2008), and (2) the subsystems of a DMU may have worse performances than the ones of another DMU, with the former being more efficient than the final (Kao and Hwang, 2010).

3.3.2.1 Structure classification

There is no only one structures that can be used to represent network systems. Charnes(1986),[2] less than ten year after the appearance of CCR, applied a basic two-stage system. In this study, he observed the effect of army recruitment in a two-process operation, allowing to detail large exercises into smaller ones. The process facilitated the identification of real impact of input factors. However, it was only more than one decade after that Färe and Grosskopf (2000) introduced the term 'network DEA'.

The formulation of the two-stage system assumes not only that the first division might have final outputs and the second division might have exogenous inputs, but also that the second division might supply the first division with some inputs. Based on concrete cases, the two-stage system can be extended to multiple stages.

In conventional network DEA, general multi-stage systems are composed of more than two divisions connected in series, when operating simultaneously, or parallel, when independently from one another. In independent operations, each division consumes a number of inputs supplied from outside and produces a number of final outputs. As stated by Kao (2017), it is also possible to develop mixed systems, resulting from a combination of assemble and disassemble divisions. Only after computing the relation between divisions is possible to identify the ones with strongest impact on system performance. All the models describe until now have a static nature, assuming that all inputs produce outputs in a given period. When dealing with interdependence in consecutive periods, dynamic systems are used as review by Fallah-Fini et al. (2014).

3.3.2.2 Literature review

Based on the literature review done and regarding the methodologies, only three of the thirty studies used network DEA applied to health care.

In a Japanese study, for example, Kawaguchi et al. (2014) measured hospitals' efficiency considering two divisions: medical examination division and administration division. While in the administration

[2] Charnes, A., Cooper, W., Golany, B., Halek, R., Klopp, G., Schmitz, E., and Thomas, D. (1986). Two-phase data envelopment analysis approaches to policy evaluation and management of army recruiting activities: trade-offs between joint services and army advertising. Technical Report 532, Center for Cybernetic Studies, University of Texas-Austin, Austin, T

division the business management is carried out, in the medical examination division the care service is provided. A Dynamic-Network DEA (DN-DEA) was applied allowing the estimation of both efficiencies, separated organizations and dynamic changes. It considers the internal heterogeneous organizations of DMUs, where divisions are mutually connected by link variables and trade internal products with each other. In addition, each DMU has carry-over variables that take into account a positive or negative factor in comparison with the previous period. Since the administrations staff does not directly engage in the production of medical services. In the case of the Black-box (BB) model, this input may correspond with the number of inpatients as an output, causing an undesirable bias in the efficiency estimation (Kawaguchi et al., 2014). Therefore, the DN-DEA model conceptually reduces bias, by both considering the multiple-step production structure and by excluding inadequate interactions between inputs and outputs.

Two year later, another study was done using the dynamic-network DEA to evaluate efficiency of public health and medical care provision in OECD countries (Ozcan and Khushalani, 2016). Analysing public health and medical care divisions independently, or the impact of public health on medical care and overall health system is essential, and only possible with DN-DEA. The DN-DEA model is similar to the Malmquist index model (Färe et al., 1994) and breaks down the overall change in efficiency into: Frontier change (change in efficiency across all health systems in the peer group due to change in innovation) and Catch-up (distance of the focus health system from the efficiency frontier). Although the Malmquist index has been used for a number of applications in health care, the DN-DEA was a relatively new model, providing the advantage of separately evaluating impact of reform on public health and medical care divisions of a health system. The two divisions used in the model were: public health and medical care, where the variables are denominated in the literature review table A.

Another study found using DN-DEA in health sector, was published in 2017 and analysed the efficiency of hospitals producing quality (Khushalani and Ozcan, 2017). The divisions chosen were the quality unit and medical-surgical care unit to establish whether these two works synergistically or there are trade-offs involved in improving the efficiency of one division to the other.

None of these studies analysed have performed an efficiency analysis with more than two division. However, in the this dissertation, the propose is three divisions. Nonetheless, the three studies reveal the importance of applying this recent methodology in order to evaluate relevance and utility for health care sector. For example, a study which performed DEA (Caballer-Tarazona et al., 2010) salient that one hospital may present as efficiency, but inefficient services might exist within the hospital.

Bottom line, it is clear that there is a knowledge gap in exploiting the potential of network DEA in the healthcare context, at least in a consistent way. Noticing, that there is no study comparing network and standard DEA. Neither there is a study applying this methodology to healthcare in Portugal.

3.4 Summary

This chapter introduced technical concepts regarding the efficiency measurement in healthcare. To evaluate hospital performance, multiple methods were introduced parallel to their strengths and weakness. Lastly, the DEA approach was investigated, including model orientation, returns-to-scale notions and recent adaptations that are starting to be used in healthcare sector.

Nevertheless, efficiency measurement is not as simple as the definition presented in 1957 by Farrell, having several aspects that need to be addressed if a functional and practical model is to be developed: (1) the organization to be analysed; (2) the outputs; (3) the inputs under consideration; (4) the external influences to the realization; and (5) the connection with the rest of the health system. The first aspect has been addressed in the second Chapter of this dissertation, so the other four aspects were the main focus on the third Chapter. Inputs and outputs consist of a major factor of efficiency measurement, since the chosen variables influence the obtained results. Understanding them allows a better development of the model in the upcoming steps.

In the following chapter the focus will be directed to explaining in detail the model conceived in this dissertation.

4

Methodology

Contents

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4.3 Summary 52

Chapter 4 lays out the reasoning behind the construction of the model formulated in the scope of this dissertation. Section 4.1 describes not only the methodologies in which the standard DEA was built on but also the network DEA methodologies. In the second section of this chapter, it is reported the implementation of its process, intrinsic procedures, and variations of the methodology due to particular characteristics.

4.1 Methodological choices

In healthcare, DEA, as stated in Subsection 3.3.1, is considered to be the leading performance evaluation approach (Hollingsworth, 2003). Although there are several methodologies to determine the DMUs that form the efficient frontier, vide supra, DEA is able to evaluate DMUs using multiple inputs to produce multiple outputs. For that reason, and since the healthcare sector is pointed out as one of the most complex, this work relied, on the first part, on a slacks-based efficiency measurement. The additive model considering VRS is the chosen model. It firstly proposed by Charnes et al. (1985). Yet, the number of variables included in the model must also be taken into consideration, since DEA is sensitive to this parameter. The lack of discrimination between efficient and inefficient DMUs often arises when there is a relatively large number of performance variables when compared to the number of DMUs. In the literature, this is often referred to as the "curse of dimensionality" (Charles et al., 2019) and will later be addressed in this chapter. In Chapter 2, the difference between private hospitals and government hospitals is defined. Private hospitals aim to attract patients to their facilities by marketing or other means and increase their organization's efficiency given their input set. In the healthcare sector, imperfect competition, constraints on finance, external influences, and regulatory constraints often result in organizations operating at an inefficient scale. Therefore, assuming a VRS is the most appropriate choice, because it will not be determined if all DMUs are operating at an optimal scale. Additionally, from a managerial perspective, the extension in which the scale of operations affects productivity is preferred (Jacobs et al., 2006). In fact, the traditional DEA model is often considered a BB model, since the internal structure of DMUs is not considered. For instance, it is unable to examine medical care divisions independently. As an extension of the BB model, network DEA accounts for divisional efficiencies, as well as overall efficiency, in a unified framework, which is applied in the second phase of this dissertation.

In this study, since it will not only applied the DEA methodology but also network DEA, it will be possible to compare their performance. By implementing DEA to analyse CUF's hospitals and the selected services, and posterior a network DEA application to analyse the same hospitals' efficiency, a more in-depth perspective is performed. This allows to (1) obtain an overall efficiency of the entire hospital; (2) determine the importance of incorporation of the services' interconnection within the hospital; and (3) measure the individual efficiency of each of the selected services.

4.1.1 Standard DEA

Radial DEA measures, which can be performed from either the input or output side, have difficulty in defining weakly efficient DMUs. Therefore, these units cannot be compared with inefficient DMUs. Additionally, input and output factors must be considered separately, resulting in inconsistent efficiency scores (Kao, 2017). Slacks-based models emerge to overcome this weakness of other DEA models, culminating in the chosen methodology for the present dissertation.

The slacks-based approach uses slacks, $(X_0 - \sum_{j=1}^{n} \lambda_j X_j, \sum_{j=1}^{n} \lambda_j Y_j - Y_0)$ to measure performance. The first effort to use slacks was proposed by Charnes et al. (1985) and the idea consists of maximizing the sum of slacks associated with the inputs,s_i^-, and outputs, s_r^+, according to Equation (4.1) and the constraints Equation (4.2-4.5).

$$\max \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \tag{4.1}$$

$$\text{s.t} \quad \sum_{j=1}^{n} \lambda_j X_{ij} + s_i^- = X_{ik}, \qquad i = 1, ..., m; \tag{4.2}$$

$$\sum_{j=1}^{n} \lambda_j Y_{rj} - s_r^+ = Y_{ik}, \qquad r = 1, ..., s; \tag{4.3}$$

$$\sum_{j=1}^{n} \lambda_j = 1, \qquad j = 1, ..., n; \tag{4.4}$$

$$\lambda_j, s_i^-, s_r^+ \geq 0, \qquad \begin{Bmatrix} j = 1, ..., n \\ r = 1, ..., s \\ i = 1, ..., m \end{Bmatrix} \tag{4.5}$$

The process is repeated n-times, once for each DMU_j, with $j = 1, ..., n$ since the model is searching for an efficient point in the production frontier that is the most distant from the DMU under analysis. By comparing each unit with all other units, the model identifies those that are operating inefficiently when compared with the other units' results. A given DMU_k is efficient if all slack variables have a value of zero. When the efficient units are determined, the efficiency frontier is defined. DMU_k's efficiency is computed using the geometric mean, according to Equation (4.6), the best fit for compounding numbers expressed in different units and introducing a certain degree of non-compensability between indicators.

$$E_k = \frac{\left(\prod_{i=1}^{m} 1 - \frac{s_i^-}{x_{ik}} \right)^{\frac{1}{m}}}{\left(\prod_{r=1}^{s} 1 + \frac{s_r^+}{y_{rk}} \right)^{\frac{1}{s}}} \tag{4.6}$$

The standard DEA model will not only be applied to the three biggest hospitals in the CUF organization, but also to three services, (1) hospitalizations (inpatient service), (2) consultations, and (3) permanent assistance, over the same period of time. According to the data, it corresponds to 60% of the full activities of the hospitals under analysis. The remaining activity is mainly found in oncology and maternity services. Each hospital will produce 36 (thirty-six) DMUs, one for each month over the past three years, 2017, 2018, and 2019. This process will allow a better comparison between using the standard DEA and the network DEA model.

4.1.2 Network DEA

In contrast to the standard DEA, where a system is considered as a BB, network DEA takes into consideration its internal structure to generate more enlightening results. The adoption of a network DEA is essential, as the complexity of a system's structure increases. Therefore, the application of this methodology in the healthcare sector is no exception, especially when hospitals composed of several services are the entities under analysis.

In the literature, measuring systems' efficiency with network DEA models has been achieved using various models. According to Kao (2014), there are nine types of models. However, they can be grouped into three (Kao, 2009), since all have a multiplier, an envelopment, and a slacks-based form. First, the independent model, which assumes that each division is an independent DMU, measuring their respective efficiencies by applying conventional DEA models. For the second type, connected models emerge as an alternative to independent models. The last category, relational models, which rest in the slacks-based form, combines the two previous concepts. The intersection of independent and connected models, allows relational models to measure system and its divisions' efficiency (Kao, 2013).

As observed in Subsection 3.3.2, network DEA is not abundantly used in the healthcare sector. However, some examples outside the healthcare sector can be found regarding the slacks-based model (see, *e.g.*, Avkiran (2009); Lin and Chiu (2013); Huang et al. (2014); Moreno and Lozano (2014)). Furthermore, since one of the main goals of this dissertation is to compare the system's performance using the DEA model versus using the network DEA, the models must have similar methodologies. Therefore, the proposed model is a slacks-based using a matrix-type structure, based on Pereira et al. (2020). The following model was subjected to some adaptations since the intermediate variables were available. No simulation of internal connections had to be done in the process.

$$\max \sum_{w=1}^{W} s_w^{(p,p')+} + \sum_{r=1}^{s} s_r^{(p)+} + \sum_{i=1}^{m} s_i^{(p)-} + \sum_{t=1}^{T} s_t^{(p',p)-} \tag{4.7}$$

$$\text{s.t}\quad \sum_{j=1}^{n} \lambda_j^{(p)} z_{wj}^{(p,p')} - s_w^{(p,p')+} = z_{wk}^{(p,p')}, \qquad \begin{Bmatrix} j = 1, ..., n \\ p = 1, ..., P \\ p' = 1, ..., P' \\ w = 1, ..., W \end{Bmatrix}; \tag{4.8}$$

$$\sum_{j=1}^{n} \lambda_j^{(p)} y_{rj}^{(p)} - s_r^{(p)+} = y_{rk}^{(p)}, \qquad \begin{Bmatrix} j = 1, ..., n \\ p = 1, ..., P \\ p' = 1, ..., P' \\ r = 1, ..., s \end{Bmatrix}; \tag{4.9}$$

$$\sum_{j=1}^{n} \lambda_j^{(p)} x_{ij}^{(p)} + s_i^{(p)-} = x_{ik}^{(p)}, \qquad \begin{Bmatrix} j = 1, ..., n \\ p = 1, ..., P \\ p' = 1, ..., P' \\ i = 1, ..., m \end{Bmatrix}; \tag{4.10}$$

$$\sum_{j=1}^{n} \lambda_j^{(p)} z_{tj}^{(p',p)} + s_t^{(p',p)-} = z_{tk}^{(p',p)}, \qquad \begin{Bmatrix} j = 1, ..., n \\ p = 1, ..., P \\ p' = 1, ..., P' \\ t = 1, ..., T \end{Bmatrix}; \tag{4.11}$$

$$\sum_{j=1}^{n} \lambda_j^{(p)} = 1, \qquad j = 1, ..., n; \tag{4.12}$$

$$\lambda_j^{(p)}, s_w^{(p,p')+}, s_r^{(p)+}, s_i^{(p)-}, s_t^{(p',p)-} \geq 0 \qquad \begin{Bmatrix} j = 1, ..., n \\ w = 1, ..., W \\ r = 1, ..., s \\ i = 1, ..., m \\ t = 1, ..., T \\ p = 1, ..., P \\ p' = 1, ..., P' \end{Bmatrix} \tag{4.13}$$

In this model, the goal is to maximize the sum of slacks, $\sum_{w=1}^{W} s_w^{(p,p')+} + \sum_{r=1}^{s} s_r^{(p)+} + \sum_{i=1}^{m} s_i^{(p)-} + \sum_{t=1}^{T} s_t^{(p',p)-}$. The process is repeated n-times, once for each DMU_j. Note that, variables $x_{ij}^{(p)}$ and $y_{rj}^{(p)}$ correspond to the inputs and outputs of the system j, respectively. These variables interact directly with the surrounding environment. The variables $z_{wj}^{(p,p')}$ and $z_{tj}^{(p',p)}$ are internal variables, the outputs of the division p to division p' and inputs from division p' to division p, respectively.

The optimal division for a generic DMU_k is also computed using the geometric mean, as expressed by Equation (4.14), while the system efficiency is measured according to Equation (4.6). Although the expression, for global efficiency, is the same for DEA and network DEA, the determination of the slacks, $s_i^{(p)-}$ and $s_r^{(p)+}$, depends on other constraints, which may result in different efficiency scores.

$$E_k = \frac{\left(\prod_{i=1}^{m} 1 - \frac{s_i^{(p)-}}{x_{ik}^{(p)}}\right)^{\frac{1}{m}} \left(\prod_{t=1}^{T} 1 - \frac{s_t^{(p',p)-}}{z_{tk}^{(p',p)}}\right)^{\frac{1}{T(p',p)}}}{\left(\prod_{w=1}^{W} 1 + \frac{s_w^{(p,p')+}}{z_{wk}^{(p,p')}}\right)^{\frac{1}{W(p,p')}} \left(\prod_{r=1}^{s} 1 + \frac{s_r^{(p)+}}{y_{rk}^{(p)}}\right)^{\frac{1}{s}}} \tag{4.14}$$

4.2 Implementation details

This section discusses the implementation details of standard and network DEA by introducing a matrix-structure for the selected variables.

In order to simplify the description, the implementation was divided into three different phases through which it was possible to achieve the models' development. Here, it will be presented the three main phases besides a short explanation of its main characteristics.

1. **Elaboration of the schematic relations.**

   In other words, it consists in defining the environment's components that enter and exit the entity under analysis. As well as the presentation of the established internal relations between the different divisions selected for that entity. The specific schematic diagram for each model will be presented in the following subsections.

   Note that the internal connections are only presented for the network DEA model since the standard DEA does not consider them.

2. **Collection of available data.**

   In the second phase of the implementation of the efficiency analysis is important to select the data that is relevant to the study. This must take into consideration the model to be implemented, the type of analysis (using physical, financial, or mixed variables), and the data's availability. The details on the selected variables will be described in Chapter 5.

3. **Model's development.**

   Introduction of the model equations, previously presented, into a program that will process its particularities. In the following subsection, it will be better detailed for each model.

Each of these phases of implementation will be described with specific information for both models.

4.2.1 Standard DEA

The first implemented model, the slacks-based DEA model, described in Subsection 4.1.1 is based on the next specifications:

1. The relation between the entity under analysis, the hospital over a specific month, and the surroundings. Some variables are introduced into the $Hospital_j$, and others are produced. The goal, since this dissertation is applying a slacks-based model, is to infer the most distant efficient point, in the production frontier, from the DMU under examination. The $Hospital_j$ is defined as efficient if the slacks' sum, $\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+$ is equal to zero.

Figure 4.2.1: Schematic diagram of $Hospital_j$ interaction with the environment.

In Figure 4.2.1, the X_{ij} correspond to the i^{th} input entry of the DMU_j in the input matrix with dimension $n \times m$ while the Y_{rj} correspond to the r^{th} output entry of the DMU_j in the output matrix with dimension $n \times r$. Note that n represent the total number of DMUs

2. Concerning the selected data, seven inputs were selected in a mixed typology to cover not only the labour contribution but also the financial one. Regarding the outputs, three were selected. As previously mentioned, when selecting the number of performance variables, it is important to apply a thumb rule to avoid the "curse of dimensionality" (Charles et al., 2019). Nunamaker (1985) was one of the first to address this problem, suggesting that the number of DMUs should be at least three times the number of inputs and outputs. However, over time, other empirical rules of thumb have arisen, from the number of DMUs being at least twice the number of inputs and outputs to being twice the product of the number inputs and the number of outputs. More recently, Cooper et al. (2000) provided guidance with the Equation (4.15), stating that the sample size is a vital issue since the model tries to estimate the average behaviour of a set of DMUs (Charles et al., 2019).

$$n \geq \max(m \times s, 3(m+s)) \tag{4.15}$$

Where n is the number of DMUs, m is the number of inputs, and s is the number of outputs. This approach is the most complete since it incorporates previous rules into one. Therefore, it is the one selected to be used in this dissertation. The verification proceeded as follows:

$$n = 36 \textit{ units (hospitals' months under analysis)} \times 3 \textit{ (number of hospitals)} = 108\ DMUs \quad (4.16)$$

$$\max(7 \textit{ inputs} \times 3 \textit{ outputs}, 3 \times (10 \textit{ (inputs+outputs)}) = \max(21, 30) = 30 \quad (4.17)$$

The number of DMUs is bigger than three times the sum of inputs and outputs, so the thumbs' empirical rule has been verified.

3. The particularities on the implementation of the slacks-based model in *Python* software consists of minimizing the inverse of the objective function, Equation (4.1), based on the constraints, Equation (4.2-4.5). By minimizing the inverse of the objective function, it is actually maximizing the slacks' sum, as described above. Only after the program performs the calculation of the maximum distance to each of the other DMUs, it will determine their efficiency based on the obtained slacks, $s_i^{(p)-}$ and $s_r^{(p)+}$, since it is based on the construction of the efficiency frontier. Note that minimization is used for the only reason that its implementation in *Python* is more direct and uses less computation power (time) to process the data.

4.2.2 Network DEA

In this subsection, the network DEA will be presented. Several details have been taken into account since it is the first time a network DEA model is implemented in the private healthcare sector.

1. After discussion with the CUF supervisor, Figure 4.2.2 represents the interconnection between the selected services within the DMU_j.

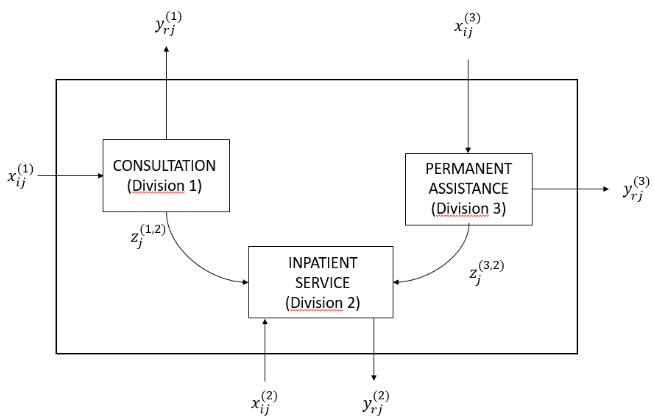

Figure 4.2.2: Schematic diagram of $Hospital_j$ interactions with the environment and between divisions 1-3, Consultation, Inpatient Service, and Permanent Assistance.

By analysing the schematic diagram, some aspects can be highlighted:

- Every division has a connection with its surroundings, as noticed by the presence of variables $x_{ij}^{(p)}$ inputs, and $x_{ij}^{(p)}$, outputs, where p corresponds to the division-*p*.
- Division 1 has an intermediate output that is provided to Division 2, $z_j^{(1,2)}$. The same relation exists between Division 3 and Division 2, $z_j^{(3,2)}$.
- Since intermediate outputs are transferred to Division 2 when the model is performing the analysis of Division 2's efficiency, they will be considered as intermediate inputs.

2. In the data selection phase, since one of the goals of this dissertation is to compare the performance analysis when using the global hospitals versus when the divisions' interconnections are taken into consideration, the following aspects were crucial. First, the total of inputs and outputs from the exterior must be the same for both models. Second, the intermediate variables should represent the relation between divisions, according to Figure 4.2.2.

   Note that, the dimensions of the inputs, or outputs, that enter, or exit, the different divisions do not have to have the same dimensions between each other. In other words, the variables' matrix $X^{(1)}$ can be $n \times m_1$, while $X^{(2)}$, can be $n \times m_2$, where $m_1 \neq m_2$. The same aspect applies to the outputs and for the intermediate variables.

3. Regarding the implementation, the main aspect is similar to the one used in the implementation of the slacks-based DEA model. There is a minimization function, inverse of the objective function according to Equation (4.7), and constraints dictated as Equation (4.8-4.13). One of the main differences is that it will be performed for each division, obtaining the corresponding slacks for each constraint resultant from the minimization. The obtained slacks, $s_i^{(p)-}$, $s_t^{(p',p)-}$, $s_w^{(p,p')+}$, and $s_r^{(p)+}$ are used for posterior efficiency calculation, as demonstrated in Equation (4.6) and (4.14).

   Although the developed model can be used for any schematic diagram, even when more interconnections are established between division, the used model has some simplifications from the general equations (4.7-4.14) for each division.

   Division 1

   The adaptation of the previous equations to the relations established in Division 1, results in the following equation:

$$\max \sum_{w=1}^{W} s_w^{(1,2)+} + \sum_{r=1}^{s} s_r^{(1)+} + \sum_{i=1}^{m} s_i^{(1)-} \tag{4.18}$$

$$\text{s.t} \quad \sum_{j=1}^{n} \lambda_j^{(1)} z_{wj}^{(1,2)} - s_w^{(1,2)+} = z_{wk}^{(1,2)}, \qquad \begin{Bmatrix} j = 1, ..., n \\ w = 1, ..., W \end{Bmatrix}; \tag{4.19}$$

$$\sum_{j=1}^{n} \lambda_j^{(1)} y_{rj}^{(1)} - s_r^{(1)+} = y_{rk}^{(1)}, \qquad \begin{Bmatrix} j = 1, ..., n \\ r = 1, ..., s \end{Bmatrix}; \tag{4.20}$$

$$\sum_{j=1}^{n} \lambda_j^{(1)} x_{ij}^{(1)} + s_i^{(1)-} = x_{ik}^{(1)}, \qquad \begin{Bmatrix} j = 1, ..., n \\ i = 1, ..., m \end{Bmatrix}; \tag{4.21}$$

$$\sum_{j=1}^{n} \lambda_j^{(1)} = 1, \qquad j = 1, ..., n; \tag{4.22}$$

$$\lambda_j^{(1)}, s_w^{(1,2)+}, s_r^{(1)+}, s_i^{(1)-} \geq 0, \qquad \begin{Bmatrix} j = 1, ..., n \\ w = 1, ..., W \\ r = 1, ..., s \\ i = 1, ..., m \end{Bmatrix} \tag{4.23}$$

Where p and p' can be substituted with the number that corresponds to the Division that is under analysis and the recipient one, respectively. Also, $s_t^{(p',p)-}$ can be removed since no intermediate inputs are introduced in Division 1. Equation (4.24) represents the formulation used to calculate Division 1's efficiency.

$$E_k = \frac{\left(\prod_{i=1}^{m} 1 - \frac{s_i^{(1)-}}{x_{ik}^{(1)}} \right)^{\frac{1}{m_1}}}{\left(\prod_{w=1}^{W} 1 + \frac{s_w^{(1,2)+}}{z_{wk}^{(1,2)}} \right)^{\frac{1}{W(p,p')}} \left(\prod_{r=1}^{s} 1 + \frac{s_r^{(1)+}}{y_{rk}^{(1)}} \right)^{\frac{1}{s_1}}} \tag{4.24}$$

The same methodology was applied for the other Divisions, resulting in the following Equations.

Division 2

In the particular case of Division 2, the intermediate variables are inputs coming from Divisions 1 and 3.

$$\max \sum_{r=1}^{s} s_r^{(2)+} + \sum_{i=1}^{m} s_i^{(2)-} + \sum_{t=1}^{t} s_t^{(p',2)-} \tag{4.25}$$

$$\textit{s.t} \quad \sum_{j=1}^{n} \lambda_j^{(2)} y_{rj}^{(2)} - s_r^{(2)+} = y_{rk}^{(2)}, \qquad \begin{Bmatrix} j = 1, ..., n \\ r = 1, ..., s \end{Bmatrix}; \tag{4.26}$$

$$\sum_{j=1}^{n} \lambda_j^{(2)} x_{ij}^{(2)} + s_i^{(2)-} = x_{ik}^{(2)}, \qquad \begin{Bmatrix} j = 1, ..., n \\ i = 1, ..., m \end{Bmatrix}; \tag{4.27}$$

$$\sum_{j=1}^{n} \lambda_j^{(1)} z_{tj}^{(p',2)} + s_t^{(p',2)-} = z_{tk}^{(p',2)}, \qquad \begin{Bmatrix} j = 1, ..., n \\ t = 1, ..., T \\ p' = 1and3 \end{Bmatrix}; \tag{4.28}$$

$$\sum_{j=1}^{n} \lambda_j^{(2)} = 1, \qquad j = 1, ..., n; \tag{4.29}$$

$$\lambda_j^{(2)}, s_r^{(2)+}, s_i^{(2)-}, s_t^{(p',2)-} \geq 0, \qquad \begin{Bmatrix} j = 1, ..., n \\ t = 1, ..., T \\ r = 1, ..., s \\ i = 1, ..., m \\ p' = 1 and 3 \end{Bmatrix} \tag{4.30}$$

$$E_k = \frac{\left(\prod_{i=1}^{m} 1 - \frac{s_i^{(2)-}}{x_{ik}^{(2)}}\right)^{\frac{1}{m_2}} \left(\prod_{t=1}^{T} 1 - \frac{s_t^{(p',2)-}}{z_{tk}^{(p',2)}}\right)^{\frac{1}{T(p',2)}}}{\left(\prod_{r=1}^{s} 1 + \frac{s_r^{(2)+}}{y_{rk}^{(2)}}\right)^{\frac{1}{s_2}}} \tag{4.31}$$

Division 3

In this Division, the equations are the same as presented in Division 1, but with the numeration in consonance with Division 3.

General Efficiency

Equation (4.6) provides hospital k's overall efficiency. Although the intermediate variables are not included in the equation, the slacks $s_i^{(p)-}$ and $s_r^{(p)+}$ are obtained from a linear program that includes the internal relations. The dimension m and s correspond, respectively, to the total number of inputs and outputs.

4.3 Summary

Chapter 4 addresses the basis of the slacks-based model, implemented to perform the DEA analysis, where it will be possible to achieve a global overview of the Hospital and Services' efficiency. On the other hand, this chapter also unveils the basis of the slacks-based network DEA model. By presenting the schematic diagrams, Figure 4.2.1 and 4.2.2, to be analysed slacks-based standard and network DEA, it was also possible to describe the variation on the implemented formulations. Now that the methodology has been explained, Chapter 5 will put the presented models to the test in the problem's framework described in Chapter 2.

5

Case study

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In the fifth chapter, the empirical application of the standard and network DEA methodology is evaluated. Based on Chapter 2, Section 5.1 provides an overview of the specific issue under analysis. The stakeholders involved in efficiency performance are presented in Section 5.2, while the description of the data, including the variables for this case study is in Section 5.3. Then, in the section 5.4, the main results and its discussion are enlightened.

5.1 Overview

As declared in Chapter 2, guaranteeing the sustainability of the health system is a concern shared by international organizations (National Academies of Sciences et al., 2018). However, this goal can only be achieved if healthcare providers engage in a coordinated action to improve resources management and cost reduction without compromising quality.

The private sector in healthcare has been growing widely since 1990, not only in Portugal but also in other countries such as the USA, United Kingdom, and Germany, serving more than half of the population in some countries (Basu et al., 2012; Kruse et al., 2018). Therefore, considering also that private entities have access to the capital market, they are incentivized to show the highest possible profit. Efficiency measurement being of extreme importance to achieve this objective since it provides a link between planning and control (Atkinson et al., 2000). Nevertheless, there has been pointed an issue in the private sector, which is the lack of published information (Footnote 8).

In Portugal, the decree of the Basic Law on Health in 1990, allowed the privatization of healthcare providers to support the SNS. However, it was only in the post-crisis that the private hospitals gained prominence to be part of hospital and clinic networks. Nonetheless, CUF is the largest private healthcare provider in Portugal founded in 1945, in Lisbon. Its stated mission focuses on becoming a prime example of clinical and human capital excellence through the ongoing growth of an integrated network of hospitals and clinics.

The CUF large dimensions result in an even larger amount of data associated with daily business operations. Therefore IM and BI are at the forefront of strategic decisions and technological innovation; without such structures, one deprives the organization's empowerment. However, data acquisition is only the first stage to analyze hospital performance, in particular the efficiency analysis.

Research related to hospital performance, in the past three decades, has been focused on measuring technical and cost efficiency through methods such as DEA and other statistical models (Jaafaripooyan et al., 2017). In short, this definition of efficiency goes through the best possible combination of inputs for reversing the maximum output. Yet the standard DEA ignores the internal connection between hospital services - and this is precisely the gap where this dissertation emerges from.

5.2 Stakeholders and their representatives

According to Dente (2014), stakeholders are actors who have interest in the matter under consideration, are affected by the issue, or can have an active or passive influence on the decision making and implementation process. These can include individuals, organizations, and networks of individuals and groups. Therefore, the first crucial step of the decision-making process is to identify the stakeholders and their objectives (Ferretti, 2016).

As previously stated in Section 2.2, countries with governments capable of using regulatory and financial tools to control health services' delivery in the public interest, normally have well-established regulation of the private sector (Brugha and Zwi, 1996). By doing so, the government is recognizing the importance of the private sector in health care (Footnote 6). Many challenges have been faced over the last two decades with the help of the private sector. For instance, fiscal space constraint arising from financial crises, changes in disease burden (especially towards chronic, noncommunicable diseases), demographic shifts, population displacement, and cases of political and economic instability (Brugha and Zwi, 2002).

Hospitals are integrated in a complex flux, which evolves the decisions of a set of differentiated stakeholders. Therefore, it does not only depend on government influence. The CUF stakeholders are the perfect example. The diversified stakeholders' actions and decisions have a huge impact on the processes' efficiency, the care provision, and the customers' experience. This last aspect is an huge component in the private sector, since private hospitals depend on the clients' satisfaction.

Therefore, CUF can be divided into three subgroups to facilitate the stakeholders' comprehension: administrators (front-office and back-office), health professions, and managers. If, on one hand, the administrative group has a role to play in the bureaucratic component and the definition of the patients' contact, scheduling processes, case handling regarding the responsible financial entities; on the other hand, health professionals are responsible for the patient journey, health monitoring and clinic cases resolution. Finally, in the managers' group, the teams have different degrees of responsibility monitoring the production, technical and operational management.

Under the knowledge that decision-making is a process that involves an enormous diversity of intervenients, the process is facilitated by the interpretation of the patient's journey, from choosing which hospital to visit, the clinic appointment, the consultation per se, and the next steps. Nonetheless, the contributions of an additional academia expert, skilled in the area of performance assessment, may facilitate the process by determining which resources must be better allocated.

5.3 Data and variables

It should be clear by now that the main goal of the case study engaged in this dissertation is to evaluate and compare the performance of the main hospitals of CUF organization, using standard DEA and network DEA. Therefore, data and sample phase were divided into three parts. Firstly, the definition of the DMUs to be analyzed. Secondly, input and output variables selection, represented by Table 5.1 and 5.2. And lastly, analysis of the data collected from the CUF database.

5.3.1 Decision-making units (DMUs)

In previous chapters, details regarding the concepts and issues related to DMUs in the DEA framework were explored. Therefore, this section has the simple purpose of highlighting the chosen DMUs. The three biggest CUF hospitals, regarding the number of clients and the number of beds, were selected as the subject of analysis. Included in these three hospitals set is found CUF Descobertas hospital (HDSC), CUF Infante Santo hospital (HCIS), and CUF Porto hospital (HPRT). In that case, if the analysis was performed with data concerning a full year, for instance, in 2019 only three DMUs would be available. In order to enlarge the number of DMU, overcome that challenge, and prevent the appearance of the "curse of dimensionality" issue, the analysis is performed over three years (2017, 2018 and, 2019) and each DMU corresponds to a different month. The DMUs under analysis are represented in Table A.2. The 108 DMU were discriminated per year, hospital, and month under evaluation. For easier table reading, the different hospitals were emphasized by colors, which are also posteriorly used. Light green for CUF Infante Santo hospital, light blue for CUF Descobertas, and light yellow for CUF Porto.

5.3.2 Standard DEA data

Initially, meeting with the coordinator from CUF was essential to define the ideal variables to measure hospitals' efficiency. The selection was based not only on Chapter 3, literature review, but also on the variables' availability in the CUF dataset. In consonance with the stakeholders, a set of seven inputs and three outputs was chosen to address this performance evaluation. The Table 5.1 contains the selected variables, indicators, and respective descriptions.

Table 5.1: Inputs and outputs, indicators, and respective descriptions for standard DEA analysis.

Type	Indicator	Description
Input (m=7)	FTE health auxiliaries, x_{1j}	Reports the full-time equivalent that indicates the health auxiliaries' workload.
	FTE nurses, x_{2j}	Reports the full-time equivalent that indicates the nurses' workload.

Table 5.1: Inputs and outputs, indicators, and respective descriptions for standard DEA analysis.

Type	Indicator	Description
Input (m=7)	FTE Doctor with a contract, x_{3j}	Reports the full-time equivalent that indicates the fixed doctors' workload.
	FTE health technicians, x_{4j}	Reports the full-time equivalent that indicates the health technicians' workload.
	FTE Doctor without a contract, x_{5j}	Reports the full-time equivalent that indicates the non-fixed doctors' workload.
	Number of beds, x_{6j}	Details the total DMU_j's number of beds.
	Operating Costs, x_{7j}	Corresponds to the total expenses (in €) of a DMU.
Output (s=3)	Inpatients, y_{1j}	Matches the number of patients leaving the inpatient service.
	Total number of clients, y_{2j}	Outlines the absolute number of clients conducted by the DMU_j.
	Hospitalizations, y_{3j}	Appraises the absolute number of medical hospitalizations.

Concerning the selected data, mixed typology is found in the selection of the inputs. The choice was purposeful to cover not only the labor contribution, FTEs inputs but also the financial one, number of beds and operational costs. Regarding the outputs, three were selected in the same typology, health services. Health outcomes were not used as output variables due to the uncertainty when it comes to select the associated division, in the posterior analysis – network DEA application. In regards to the application of the standard DEA to each service, the variables used are described in Table 5.2. Nevertheless, since the objective is to compare the results with the ones posteriorly, both variable sets must be in agreement. Therefore, six inputs and two outputs for Division 1; seven inputs and one output for Division 2, six inputs and two outputs for Division 3.

The variables are not discriminated during the development of this thesis due to the fact that CUF is a part of the private sector; therefore, competition is an important factor. The security of the CUF organization had to be respected during the following explanations.

5.3.3 Network DEA data

When defining the variables of network DEA, it is necessary not only to take into consideration variables that connect between divisions but also the relation with the variables used in the standard DEA. Therefore, Table 5.2, represents the selected variables, indicators, respective descriptions, and assigned divisions.

Table 5.2: Inputs, intermediate variables, and outputs, indicators, respective descriptions, and correspondent division for network DEA analysis.

Type	Indicator	Description	Division
Inputs	FTE health auxiliaries, x_{1j}	Reports the full-time equivalent that indicates the health auxiliaries' workload.	1-3
	FTE nurses, x_{2j}	Reports the full-time equivalent that indicates the nurses' workload.	1-3
	FTE Doctor with a contract, x_{3j}	Reports the full-time equivalent that indicates the fixed doctors' workload.	1-3
	FTE health technicians, x_{4j}	Reports the full-time equivalent that indicates the health technicians' workload.	1 and 2
	FTE Doctor without a contract, x_{5j}	Reports the full-time equivalent that indicates the non-fixed doctors' workload.	1-3
	Number of beds, x_{6j}	Details the total DMU_j's number of beds.	2 and 3
	Operating Costs, x_{7j}	Corresponds to the total expenses (in €) of a DMU.	1-3
Intermediate variables	Number of episodes, z_{1j}	Outlines the absolute number of hospitalization episodes that result from consultation or permanent assistance	$1 \rightarrow 2$ and $3 \rightarrow 2$
	Number of patients, z_{2j}	Corresponds to the absolute number of patients who move from permanent assistance to hospitalization division	$3 \rightarrow 2$
Outputs	Inpatients, y_{1j}	Matches the number of patients leaving the inpatient service.	1 and 3
	Total number of clients, y_{2j}	Outlines the absolute number of clients conducted by the DMU_j.	1-3
	Hospitalizations, y_{3j}	Appraises the absolute number of medical hospitalizations.	2

For reference, some notes regarding the intermediate variables, intermediate outputs from Division 1 or Division 2 correspond to intermediate inputs for Division 3. For instance, $z_{1j}^{(1,2)} = z_{1j}^{(2,1)}, z_{1j}^{(3,2)} = z_{1j}^{(2,3)}$ and $z_{2j}^{(3,2)} = z_{2j}^{(2,3)}$.

5.4 Results and discussion

The application of the two models, slacks-based standard and network DEA resulted in the determination of the efficiency score of the previously presented DMUs. In this section, the results as well as their discussion will be present.

5.4.1 Model solving

Both slacks-based standard DEA and network DEA were implemented in *Python*, using a numpy package to manage the numerical variables. The calculation of the efficiency was divided into five functions:

_efficiency, _target, _constraints, _ieqconstraints, and _optimize. The first 4 functions were described with the presentation of the respective equations in Chapter 4. As for the _optimize function, which relies on previous equations, a scipy function is used *Scipy.optimize.fmin_slsqp* method is the one selected to perform the optimization. This function, implemented by Dieter Kraft [1] in scipy, does the minimization using sequential least squares programming. The model is estimated by minimizing the _target function for each unit, subject to the constraints calculated in _constrains and _ieqconstrains functions.

As previously stated, the model is optimized unit-by-unit, with slacks randomly initialized, with a uniform distribution between -0.5 and 0.5. After each slack is optimized the algorithm converges, and the efficiency of each unit is calculated and saved.

5.4.2 Standard DEA results

Table 5.3 contains the efficiency scores obtained when the slack-based DEA was applied. Note that all efficient DMUs are represented in green color, in the efficiency score column, while the partial innefficient are represented in yellow. From the table observation, the clear comment resultant is that not all DMUs are efficient.

Global Efficiency

In 108 DMUs, 63 establish the efficiency frontier, which corresponds to 58.3% of total DMUs. Another direct observation is that the majority of global efficient DMUs are found in the CUF Porto hospital.

Table 5.3: Efficiency Scores when standard DEA is applied, according to the DMU description. Specification of the divisions' efficiency using a color legend, green for efficiency division and yellow for inefficient

DMU description	Division	Efficiency Score	DMU description	Division	Efficiency Score	DMU description	Division	Efficiency Score
HCIS_Jan 2017	1 2 3	1.00	HCIS_Jan 2018	1 2 3	0.96	HCIS_Jan 2019	1 2 3	1.00
HCIS_Feb 2017	1 2 3	1.00	HCIS_Feb 2018	1 2 3	0.94	HCIS_Feb 2019	1 2 3	1.00
HCIS_Mar 2017	1 2 3	1.00	HCIS_Mar 2018	1 2 3	1.00	HCIS_Mar 2019	1 2 3	1.00
HCIS_Abr 2017	1 2 3	1.00	HCIS_Abr 2018	1 2 3	0.94	HCIS_Abr 2019	1 2 3	1.00

[1] https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.fmin_slsqp.html, acessed at 10/09/2020

Table 5.3: Efficiency Scores when standard DEA is applied, according to the DMU description. Specification of the divisions' efficiency using a color legend, green for efficiency division and yellow for inefficient

DMU description	Division	Efficiency Score	DMU description	Division	Efficiency Score	DMU description	Division	Efficiency Score
HCIS_May 2017	1 2 3	1.00	HCIS_May 2018	1 2 3	0.95	HCIS_May 2019	1 2 3	1.00
HCIS_Jun 2017	1 2 3	1.00	HCIS_Jun 2018	1 2 3	0.87	HCIS_Jun 2019	1 2 3	1.00
HCIS_Jul 2017	1 2 3	1.00	HCIS_Jul 2018	1 2 3	0.87	HCIS_Jul 2019	1 2 3	1.00
HCIS_Aug 2017	1 2 3	1.00	HCIS_Aug 2018	1 2 3	1.00	HCIS_Aug 2019	1 2 3	1.00
HCIS_Sep 2017	1 2 3	1.00	HCIS_Sep 2018	1 2 3	0.87	HCIS_Sep 2019	1 2 3	0.75
HCIS_Oct 2017	1 2 3	0.97	HCIS_Oct 2018	1 2 3	0.94	HCIS_Oct 2019	1 2 3	0.93
HCIS_Nov 2017	1 2 3	1.00	HCIS_Nov 2018	1 2 3	0.98	HCIS_Nov 2019	1 2 3	1.00
HCIS_Dec 2017	1 2 3	0.90	HCIS_Dec 2018	1 2 3	0.83	HCIS_Dec 2019	1 2 3	1.00
HDSC_Jan 2017	1 2 3	0.84	HDSC_Jan 2018	1 2 3	0.94	HDSC_Jan 2019	1 2 3	0.81
HDSC_Feb 2017	1 2 3	0.83	HDSC_Feb 2018	1 2 3	0.77	HDSC_Feb 2019	1 2 3	0.84
HDSC_Mar 2017	1 2 3	1.00	HDSC_Mar 2018	1 2 3	1.00	HDSC_Mar 2019	1 2 3	0.91
HDSC_Apr 2017	1 2 3	0.82	HDSC_Apr 2018	1 2 3	0.82	HDSC_Apr 2019	1 2 3	0.84
HDSC_May 2017	1 2 3	0.92	HDSC_May 2018	1 2 3	1.00	HDSC_May 2019	1 2 3	0.90

Table 5.3: Efficiency Scores when standard DEA is applied, according to the DMU description. Specification of the divisions' efficiency using a color legend, green for efficiency division and yellow for inefficient

DMU description	Division	Efficiency Score	DMU description	Division	Efficiency Score	DMU description	Division	Efficiency Score
HDSC_Jun 2017	1, 2, 3	0.76	HDSC_Jun 2018	1, 2, 3	0.85	HDSC_Jun 2019	1, 2, 3	1.00
HDSC_Jul 2017	1, 2, 3	0.83	HDSC_Jul 2018	1, 2, 3	0.79	HDSC_Jul 2019	1, 2, 3	0.84
HDSC_Aug 2017	1, 2, 3	0.61	HDSC_Aug 2018	1, 2, 3	1.00	HDSC_Aug 2019	1, 2, 3	0.73
HDSC_Sep 2017	1, 2, 3	0.90	HDSC_Sep 2018	1, 2, 3	0.83	HDSC_Sep 2019	1, 2, 3	0.79
HDSC_Oct 2017	1, 2, 3	0.91	HDSC_Oct 2018	1, 2, 3	0.88	HDSC_Oct 2019	1, 2, 3	1.00
HDSC_Nov 2017	1, 2, 3	0.94	HDSC_Nov 2018	1, 2, 3	1.00	HDSC_Nov 2019	1, 2, 3	0.90
HDSC_Dec 2017	1, 2, 3	1.00	HDSC_Dec 2018	1, 2, 3	1.00	HDSC_Dec 2019	1, 2, 3	0.86
HPRT_Jan 2017	1, 2, 3	1.00	HPRT_Jan 2018	1, 2, 3	1.00	HPRT_Jan 2019	1, 2, 3	1.00
HPRT_Feb 2017	1, 2, 3	1.00	HPRT_Feb 2018	1, 2, 3	1.00	HPRT_Feb 2019	1, 2, 3	1.00
HPRT_Mar 2017	1, 2, 3	1.00	HPRT_Mar 2018	1, 2, 3	1.00	HPRT_Mar 2019	1, 2, 3	1.00
HPRT_Apr 2017	1, 2, 3	1.00	HPRT_Apr 2018	1, 2, 3	1.00	HPRT_Apr 2019	1, 2, 3	1.00
HPRT_May 2017	1, 2, 3	1.00	HPRT_May 2018	1, 2, 3	1.00	HPRT_May 2019	1, 2, 3	1.00
HPRT_Jun 2017	1, 2, 3	1.00	HPRT_Jun 2018	1, 2, 3	1.00	HPRT_Jun 2019	1, 2, 3	0.99

Table 5.3: Efficiency Scores when standard DEA is applied, according to the DMU description. Specification of the divisions' efficiency using a color legend, green for efficiency division and yellow for inefficient

DMU description	Division	Efficiency Score	DMU description	Division	Efficiency Score	DMU description	Division	Efficiency Score
HPRT_Jul 2017	1	1.00	HPRT_Jul 2018	1	1.00	HPRT_Jul 2019	1	1.00
	2			2			2	
	3			3			3	
HPRT_Aug 2017	1	1.00	HPRT_Aug 2018	1	1.00	HPRT_Aug 2019	1	1.00
	2			2			2	
	3			3			3	
HPRT_Sep 2017	1	1.00	HPRT_Sep 2018	1	0.77	HPRT_Sep 2019	1	0.96
	2			2			2	
	3			3			3	
HPRT_Oct 2017	1	1.00	HPRT_Oct 2018	1	0.93	HPRT_Oct 2019	1	1.00
	2			2			2	
	3			3			3	
HPRT_Nov 2017	1	1.00	HPRT_Nov 2018	1	1.00	HPRT_Nov 2019	1	1.00
	2			2			2	
	3			3			3	
HPRT_Dec 2017	1	1.00	HPRT_Dec 2018	1	1.00	HPRT_Dec 2019	1	1.00
	2			2			2	
	3			3			3	

After performing the standard DEA analysing the global efficiency for each Hospital in a particular month, it was also performed for each Division. The results are also included in Table 5.3.

Division 1

There were identified 48 efficient DMUs, with any particular geographic or temporal distribution.

Division 2

For the division 2, 46 DMUs are classified as efficient without a particular distribution.

Division 3

In regards to division 3, 21 DMUs were identified as efficient with no particular geographic or temporal distribution. In comparison with the other divisions, Division 3 is the less efficient. This result is understandable since permanent assistance is being evaluated. It corresponds to the most dependent on external variables, like national health in that moment of the year. For example, in Porto in December of 2019, the number of colds could have been bigger than usual, increasing the number of consultations. Nevertheless, it is important to include this division since with a strategy it is possible to predict with some error the number of episodes expected each month, better defining the necessary resources.

A comparative analysis of Table 5.3 shows that DMUs are progressively less efficient, from global efficiency which identified 63 efficient DMU to Division 3, which only identified 21 efficient DMUs of a total of 108 DMUs.

5.4.3 Network DEA results

The Table 5.4 contains the efficiency scores of all DMUs regarding each service, as well as the global efficiency of DMUs according to quartiles. As previously done, the divisions' efficiency is represented in green color, while the inefficient are in yellow. The same is applied for the global efficiency, efficient DMUS are represented in green in the efficiency score column. In the 108 DMUS, 20 of them are globally efficient, which corresponds to 18.5%. Therefore, when the internal correlations between divisions are taken into consideration the efficiency decreases. Most of the efficient DMUS, when network DEA is applied are found in CUF Infante Santo which is different from the result previously obtained. Nevertheless, not all of these DMUs are efficient in every service, and not every inefficient DMU is inefficient in every service. From the 20 globally efficient DMUs, eight are not efficient in every service under analysis, viz., HCIS_Abril2017, HCIS_Aug2017, HCIS_Nov2017, HDSC_May2017, HCIS_Jun2018, HCIS_Aug2018, and HDSC_May2018. Although all of them are efficient in at least 2 of the 3 divisions under analysis. Despite the services of these DMUs have worse performance than the other ones, globally, hospitals are also seen as efficient, as previously observed by (Kao and Hwang, 2010). There are also no reported cases of DMUS that are globally inefficient, but efficient in every service. Moreover, since each division is analyzed as an independent DMU with internal variables, DMUs which are globally inefficient, but present an efficient service may still act as a benchmark for that particular service.

Table 5.4: Efficiency Scores when network DEA is applied, according to the DMU description. Specification of the divisions' efficiency using a color legend, green for efficiency division and yellow for inefficient

DMU description	Division	Efficiency Score	DMU description	Division	Efficiency Score	DMU description	Division	Efficiency Score
HCIS_Jan 2017	1 2 3	0.99	HCIS_Jan 2018	1 2 3	1.00	HCIS_Jan 2019	1 2 3	0.92
HCIS_Feb 2017	1 2 3	0.97	HCIS_Feb 2018	1 2 3	0.98	HCIS_Feb 2019	1 2 3	0.91
HCIS_Mar 2017	1 2 3	1.00	HCIS_Mar 2018	1 2 3	1.00	HCIS_Mar 2019	1 2 3	0.93
HCIS_Abr 2017	1 2 3	1.00	HCIS_Abr 2018	1 2 3	0.97	HCIS_Abr 2019	1 2 3	0.92
HCIS_May 2017	1 2 3	1.00	HCIS_May 2018	1 2 3	0.95	HCIS_May 2019	1 2 3	0.91

Table 5.4: Efficiency Scores when network DEA is applied, according to the DMU description. Specification of the divisions' efficiency using a color legend, green for efficiency division and yellow for inefficient

DMU description	Division	Efficiency Score	DMU description	Division	Efficiency Score	DMU description	Division	Efficiency Score
HCIS_Jun 2017	1 2 3	1.00	HCIS_Jun 2018	1 2 3	1.00	HCIS_Jun 2019	1 2 3	0.92
HCIS_Jul 2017	1 2 3	0.97	HCIS_Jul 2018	1 2 3	0.91	HCIS_Jul 2019	1 2 3	0.97
HCIS_Aug 2017	1 2 3	1.00	HCIS_Aug 2018	1 2 3	1.00	HCIS_Aug 2019	1 2 3	0.91
HCIS_Sep 2017	1 2 3	0.99	HCIS_Sep 2018	1 2 3	0.97	HCIS_Sep 2019	1 2 3	0.91
HCIS_Oct 2017	1 2 3	1.00	HCIS_Oct 2018	1 2 3	0.99	HCIS_Oct 2019	1 2 3	0.99
HCIS_Nov 2017	1 2 3	1.00	HCIS_Nov 2018	1 2 3	0.95	HCIS_Nov 2019	1 2 3	0.93
HCIS_Dec 2017	1 2 3	0.99	HCIS_Dec 2018	1 2 3	0.96	HCIS_Dec 2019	1 2 3	0.92
HDSC_Jan 2017	1 2 3	1.00	HDSC_Jan 2018	1 2 3	1.00	HDSC_Jan 2019	1 2 3	0.98
HDSC_Feb 2017	1 2 3	0.97	HDSC_Feb 2018	1 2 3	0.85	HDSC_Feb 2019	1 2 3	0.93
HDSC_Mar 2017	1 2 3	0.99	HDSC_Mar 2018	1 2 3	0.99	HDSC_Mar 2019	1 2 3	0.92
HDSC_Apr 2017	1 2 3	0.96	HDSC_Apr 2018	1 2 3	0.85	HDSC_Apr 2019	1 2 3	0.97
HDSC_May 2017	1 2 3	1.00	HDSC_May 2018	1 2 3	1.00	HDSC_May 2019	1 2 3	0.94
HDSC_Jun 2017	1 2 3	0.98	HDSC_Jun 2018	1 2 3	0.95	HDSC_Jun 2019	1 2 3	0.91

Table 5.4: Efficiency Scores when network DEA is applied, according to the DMU description. Specification of the divisions' efficiency using a color legend, green for efficiency division and yellow for inefficient

DMU description	Division	Efficiency Score	DMU description	Division	Efficiency Score	DMU description	Division	Efficiency Score
HDSC_Jul 2017	1 2 3	0.93	HDSC_Jul 2018	1 2 3	0.95	HDSC_Jul 2019	1 2 3	0.95
HDSC_Aug 2017	1 2 3	0.92	HDSC_Aug 2018	1 2 3	0.92	HDSC_Aug 2019	1 2 3	0.84
HDSC_Sep 2017	1 2 3	0.97	HDSC_Sep 2018	1 2 3	0.92	HDSC_Sep 2019	1 2 3	0.99
HDSC_Oct 2017	1 2 3	1.00	HDSC_Oct 2018	1 2 3	0.88	HDSC_Oct 2019	1 2 3	1.00
HDSC_Nov 2017	1 2 3	1.00	HDSC_Nov 2018	1 2 3	0.89	HDSC_Nov 2019	1 2 3	0.98
HDSC_Dec 2017	1 2 3	0.98	HDSC_Dec 2018	1 2 3	0.90	HDSC_Dec 2019	1 2 3	0.95
HPRT_Jan 2017	1 2 3	0.86	HPRT_Jan 2018	1 2 3	0.96	HPRT_Jan 2019	1 2 3	0.98
HPRT_Feb 2017	1 2 3	0.82	HPRT_Feb 2018	1 2 3	0.93	HPRT_Feb 2019	1 2 3	0.96
HPRT_Mar 2017	1 2 3	0.78	HPRT_Mar 2018	1 2 3	0.98	HPRT_Mar 2019	1 2 3	0.88
HPRT_Apr 2017	1 2 3	0.95	HPRT_Apr 2018	1 2 3	0.95	HPRT_Apr 2019	1 2 3	0.93
HPRT_May 2017	1 2 3	0.85	HPRT_May 2018	1 2 3	0.95	HPRT_May 2019	1 2 3	0.98
HPRT_Jun 2017	1 2 3	0.98	HPRT_Jun 2018	1 2 3	0.82	HPRT_Jun 2019	1 2 3	0.92
HPRT_Jul 2017	1 2 3	0.93	HPRT_Jul 2018	1 2 3	0.96	HPRT_Jul 2019	1 2 3	0.94

Table 5.4: Efficiency Scores when network DEA is applied, according to the DMU description. Specification of the divisions' efficiency using a color legend, green for efficiency division and yellow for inefficient

DMU description	Division	Efficiency Score	DMU description	Division	Efficiency Score	DMU description	Division	Efficiency Score
HPRT_Aug 2017	1	1.00	HPRT_Aug 2018	1	0.90	HPRT_Aug 2019	1	0.90
	2			2			2	
	3			3			3	
HPRT_Sep 2017	1	0.97	HPRT_Sep 2018	1	0.96	HPRT_Sep 2019	1	0.92
	2			2			2	
	3			3			3	
HPRT_Oct 2017	1	1.00	HPRT_Oct 2018	1	0.98	HPRT_Oct 2019	1	0.98
	2			2			2	
	3			3			3	
HPRT_Nov 2017	1	0.97	HPRT_Nov 2018	1	0.95	HPRT_Nov 2019	1	0.92
	2			2			2	
	3			3			3	
HPRT_Dec 2017	1	0.93	HPRT_Dec 2018	1	0.92	HPRT_Dec 2019	1	0.93
	2			2			2	
	3			3			3	

Although the number of globally efficient DMUs is smaller for network DEA than for standard DEA, in partial efficiency, more DMUs are identified for network DEA. For instance, Division 1 presents 52 efficient DMUs, Division 2 shows 73 efficient DMUs, and in Division 3, 32 DMUs.

5.4.4 Fundamental statistics

After obtaining the DEA results for both methodologies, it is important to address its fundamental statistics. The results of the selected indicators are displayed in Table 5.5 and 5.6, for partial and global efficiency, respectively. Note that the maximum was not presented as an indicator because it would be 1.00, the score when the DMU is efficient, for all the sets of analysis.

Table 5.5: Indicator's basic statistics, for partial efficiency.

Indicator	$\bar{x}$	σ	CV	Min	Q_1	Q_2	Q_3	IQR
Stan_div_1	0.93	0.09	9.61	0.67	0.87	0.99	1.00	0.13
Stan_div_2	0.92	0.12	13.51	0.38	0.88	0.98	1.00	0.12
Stan_div_3	0.79	0.16	19.87	0.44	0.65	0.78	0.93	0.28
Net_div_1	0.90	0.13	14.11	0.46	0.83	0.98	1.00	0.17
Net_div_2	0.92	0.13	14.53	0.58	0.85	1.00	1.00	0.15
Net_div_3	0.72	0.26	35.37	0.10	0.52	0.72	1.00	0.48

Regarding the arithmetic average within each methodology, it is observed a decreased according, as expected, with the decrease of efficient DMUs. Nevertheless, Division 2 using network DEA is the one which identified more efficient DMUs and does not present the biggest arithmetic average. Therefore, it is important to analyse other variables, and what is observe is that the standard deviation is bigger than in Division 1 with Standard DEA (higher arithmetic average). Another relevant parameter is the coefficient of variation, which is below 25%, the reference literature in all indicators but Net_div_3. Accordingly, these indicators do not demonstrate significant heterogeneity as then confirmed by the interquartile range. The biggest heterogeneity, in both methodologies, is found in Division 3, which is, as previously mentioned, to the most depend on external variables. However, the coefficient variations are generally bigger when network DEA is applied.

Table 5.6: Indicator's basic statistics, for global efficiency

Indicator	$\overline{x}$	σ	CV	Min	Q_1	Q_2	Q_3	IQR
Standard	0.93	0.09	9.61	0.67	0.87	0.99	1.00	0.13
Network	0.92	0.12	13.51	0.38	0.88	0.98	1.00	0.12

Furthermore, for the global efficiency, although the coefficient of variation is bigger and the minimum is lower for the network DEA, the interquartile range is similar between methodologies, which means that the statistical dispersion is identical for both.

5.4.5 Findings

The comparison between the two models is one of the main goals of the present thesis, so a detailed comparison between division analysis was firstly performed.

Partial Efficiency

In order to better compare the services analyzed by both methodologies, a table was created with the number of identified efficient DMUs for each division.

Table 5.7: Number of efficient units for each division for both methodologies, standard DEA and network DEA

Division	# efficient DMUs using standard DEA	# efficient DMUs using network DEA	Error
Division 1	48	52	4 units
Division 2	46	73	27 units
Division 3	20	32	12 units

The first observation that comes from the results presented in Table 5.7 is that network DEA evaluates more DMUs as efficient than the application of standard DEA. However, it is important to do a deeper analysis. By closely comparing each DMUs' efficiency score for each division, it was observed that all

DMUs identified as efficient when applying standard DEA were also efficient when network DEA was used. Another aspect that may be important to stress is that Division 2, the only division that had a connection with more than one division, more specifically, 3 internal variables were entering it, is the one which presents the biggest differences in the number of DMUs identified as efficient between both methodologies. Even more interesting, is that Division 1, which is the one with less internal variables is the one that has the lowest difference between standard and network DEA.

Global Efficiency

Table 5.8: The number of globally efficient units for both methodologies, standard DEA, and network DEA.

# efficient DMUs using standard DEA	# efficient DMUs using network DEA	Difference
63	20	43 units

The obvious observation is that the difference between standard DEA and network DEA is even bigger when the global analysis is being done. However, that is not the only interesting observation. For instance, contrary to the partial efficiency, for global efficiency, the standard DEA identifies more DMUs as efficient than network DEA. Another interesting aspect was noticed when a deeper analysis was performed. Not all the DMUs which are efficient for standard DEA are efficient for network DEA, and the other way around was also verified. The Table A.3 summarizes the DMUs which are efficient when one methodology is applied but not when the other is.

Reflecting on the information provided, there are 51 DMUs that are only recognized as efficient when applying standard DEA and eight DMUs which are only identified as efficient when network DEA is used. From all the DMUs identified as efficient, only 12 DMUs are identified as efficient for both methodologies.

Concluding, the introduction of internal connections as an efficiency factor results not only in the decrease of efficiency but may also result in its increase, depending on the internal connections significance. Therefore, the study of the impact of specific connections in the determination of global efficiency appears to be an interesting topic for future studies.

5.5 Business viewpoint

The unbiased efficiency analysis, based on objective data, is crucial. Nonetheless, this analysis can never be dissociated from the business analysis. By including the healthcare business' point of view, the full scope of efficiency analysis will be covered.

The following analysis will be divided according to the group of hospitals under investigation. Furthermore, since the relevant historical aspects seem to be aligned with the results obtained when the network DEA was applied, it will be the methodology considered in the analysis of this dissertation's section. Through examining the hospital's history from the previous three years, it is possible to find some situations which influenced the efficiency results.

For instance, in CUF Infante Santo, represented in light green, even though there were identified eigth efficient DMUs in 2017 and four efficient DMUs in 2018, no efficient DMU is determined in 2019. The decrease of efficiency in 2019 is concordant with the reinforcement of medical staff, which will not only increase the FTEs but also the associated hospital's costs. This expansion of medical staff is a normal procedure to integrate human resources in the organization dynamic before the opening of new hospitals. Afterward, CUF Tejo, a new hospital, would inaugurate in 2020. Therefore, the incrementation of these two input variables, without the increase in the number of consultations results in the observed decrease in efficiency.

As for CUF Descobertas, represented in light blue, the opening of building 2 in July of 2018, involved the upgrade of several medical specializations. Therefore, it was also verified an increase in FTEs which takes some time to be reflected in the increase of clients. These variations can justify the non-existed efficient DMUs in the second half of 2018. However, there is not only an increase in the FTEs but also the costs associated with the equipment of the new building. Only more than one year later, in October of 2019, there is an efficient DMU identified in CUF Descobertas. Nonetheless, there is another historical moment, which may be relevant to point out, the legionella outbreak in February of 2018 . CUF Descobertas suffered an outbreak of legionella. No doubt it affected the efficiency not only due to the decrease of clients and hospitalizations but also the cost of disinfestation and medical staff. Measures essencial for client safety. These fluctuations are detected in the decrease of efficiency score in February of 2018.

Lastly, CUF Porto, represented in a light yellow, which seems to be the least efficient when network DEA is applied, although the most efficient for standard DEA, has almost no historical facts to justify this inefficiency. There is only the accreditation of the International Joint Commission in June of 2018. This accreditation may have led to some entropy in the hospital's organization resulting in inefficiency. Nevertheless, this fact would not justify the non-existent efficient DMUs since October 2017. Along with that, further investigation in the CUF Porto efficiency must be performed.

To finalize, there was also identified general modification, for the three hospitals under analysis. For instance, in 2018, the CUF organization, invested in the renovation of hospital equipment which influenced the total costs from thereon. This appears to be concordant with the decrease of efficiency throughout the years. In 2019, to add to this cost incrementation, there was also officialized the nursing career, resulting in retention policy and therefore an increase of nurses' FTEs. Although this has an incredible effect on the life quality of these professionals, it will also result in more inputs that are not immediately reflected in the increase of outputs, decreasing the immediate global efficiency.

The present business analysis provided insightful justification for the detected inefficiency. By considering not only the data but also the business point of view, the obtained results were supported, and even more, the network DEA was valued. As the inclusion of the hospital fluxes seems to be in better concordance with the environmental facts. Network DEA provides a holistic view of the hospital dynamics.

5.6 Summary

In essence, for divisions analysis, standard DEA tends to estimate a lower quantity of efficient DMUs than when the network DEA is applied. However, all DMUs which are efficient for standard DEA are also efficient for network DEA methodology. In contrary to global efficiency where not only the standard DEA estimates a higher quantity of efficient DMUs than the network DEA; but also the DMUs which are efficient for networks DEA are not always efficient for standard DEA. For instance, from the 20 DMUs which are identified as efficient when network DEA is applied, only 12 are simultaneously efficient for both methodologies.

Overall there is no pattern, geographical or temporal, in the identification of efficient DMUs, neither within the chosen methodology, standard DEA and network DEA, nor within partial or global efficiency. Nevertheless, in order to better visualize the DMUs display, it was constructed a schematic representation of the efficient DMUs. In this, each line corresponds to one hospital, and each rectangle to one DMU, so one of the thirty-six months studied. Furthermore, for DMUs that are efficient for both methodologies, the rectangle is in dark green with dotted outline, while DMUs efficient for only one methodology, the rectangle is in orange with dashed outline.

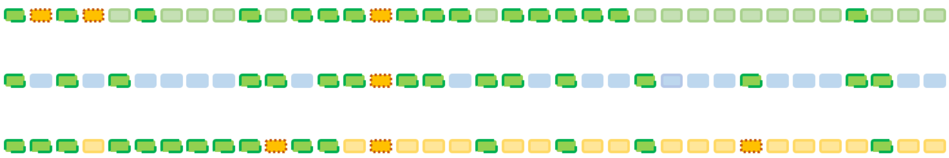

Figure 5.6.1: . Schematic representation of the DMUs, which correspond to division 1, which are efficient for both methodologies, in green with dotted outline, and for one of the methodologies, in orange with dashed outline.

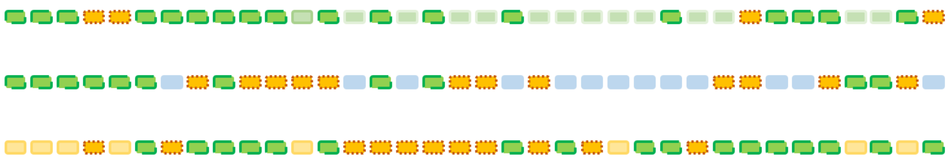

Figure 5.6.2: Schematic representation of the DMUs, which correspond to division 2, which are efficient for both methodologies, in green with dotted outline, and for one of the methodologies, in orange with dashed outline.

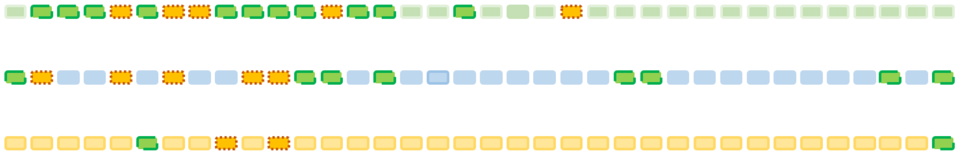

Figure 5.6.3: Schematic representation of the DMUs, which correspond to division 3, which are efficient for both methodologies, in green with dotted outline, and for one of the methodologies, in orange with dashed outline.

From the observation of these three schematic figures, Figure 1, 2, and 3 results the conclusion that most efficient DMUs, for both methodologies and the three services, are found in the hospital represented in light green, CUF Infante Santo. Regarding consultations, division 1, the hospital which presents more inefficient DMUs is CUF Descobertas, represented in blue. In respect to hospitalization, divisions 2, the least efficient (with more inefficient DMUs) hospital is also CUF Infante Santo, although hospitals are operating closely, with only one less DMU identified as inefficient for CUF Descobertas (13 DMUs). As for permanent assistance (AP), division 3, the least efficient hospital is CUF Porto, represented in yellow.

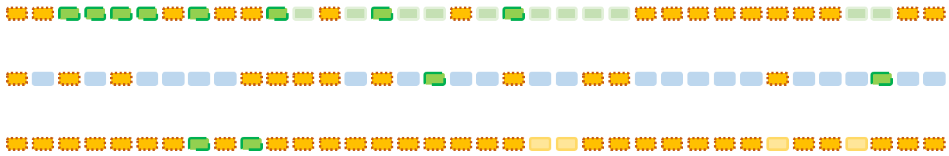

Figure 5.6.4: Schematic representation of the DMUs which are globally efficient for both methodologies, in green with dotted outline, and for one of the methodologies, in orange with dashed outline.

When it comes to analyzing global efficiency, the hospital which presents the least quantity of inefficient DMUs is CUF Porto (light blue). However, if both methodologies are considered, CUF Infante Santo (light green) is the one that presents more efficient DMUs.

6

Conclusions and future remarks

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6.1 Achievements

The main objective, as stated in Chapter 1 of this dissertation, was to develop two models, standard DEA and network DEA, to perform its comparison. Although efficiency measurement is an important step to find resources optimization, and DEA is the most used methodology in the health sector (Chapter 2) (Hollingsworth, 2016; Kao, 2017), no comparison between these two methodologies was found in literature, as verified through the extensive literature review conducted in Chapter 3. The additive model considering variable returns to scale (VRS), which is the chosen model, was initially proposed by Charnes et al. (1985). However, it had to suffer some adaptation according to system and variables under analysis. Furthermore, the network DEA, based on Pereira et al. (2020), is also a slacks-based model since model similarities are fundamental to compare models' results. Every detail in methodology implementation is unveiled in Chapter 4. After defining, develop and implement, both models were applied to the three major hospitals of CUF, the largest private healthcare provider in Portugal. Chapter 5 disclose the definition of three services which are used as hospital's divisions in the network DEA, consultations, hospitalization, and permanent assistance. Nevertheless, the selection of these services is based on their hospital representation, since in conjugation the services correspond to 60% of total activity. Moreover, the case study allowed not only the analysis of the robustness of slacks-based network DEA; but also the obtaining of efficiency scores for partial (services) and global efficiency, which can assist the decision-making when it comes to reducing inputs and increase the output.

Therefore, the development of the slacks-based DEA and network DEA yield the first comparison of these two methodologies. Thus, the intention was to introduce the less uncontrolled variables possible, creating the models based in the same typology, additive model, and maintaining the inputs and outputs of the hospital through both methodologies. However, although for partial efficiency the DMUs identified as efficient are fewer for standard DEA than for network DEA in every division, the same is not verified for global efficiency where more DMUs were identified as efficient for standard DEA, 63 efficient DMUs in comparison with 20 efficient DMUs for network DEA. Moreover, in partial efficiency, the DMUs identified as efficient for the standard DEA were also efficient for the network DEA, which is not the case for global efficiency. Another element to consider regarding the partial efficiency analysis is that Division 2, which is the only division that established connections with more than one division, had the biggest difference between methodologies concerning the number of efficient DMUs identified. Even more, Division 1, which only had one internal output had the lowest difference between standard and network DEA. Essentially, if on one hand these conclusions demonstrated the dependency of efficiency on the internal connections between division, as observed in the partial efficiency results; on the other hand, it is incomplete for the formulation of final conclusions regarding the differences between standard and network DEA for global efficiency. Nevertheless, this dissertation fills the knowledge gap in the literature respecting to the comparison of these two methodologies and provides healthcare manager and policymakers ability to distinguish methodology results and better support decision-making.

In resume, although the efficient DMUs are not exactly the same, in general the network DEA is more demanding in the identification of efficient DMUs, pointing out only 18.2% in a total of 108 DMUs instead of the 58.3% pointed by standard DEA. For all of that, network DEA appears to be a more enlightening methodology. It includes internal system fluxes and consequently separates partial and global efficiency scores, providing a holistic view of the hospital dynamics. Nevertheless, future researches, as better explained in subsection 6.4, should include variations and incorporations which were left outside of this dissertation.

Therefore, at the core, the goals set at the beginning of this dissertation were doubtless achieved, since:

- The efficiency analysis using standard and network DEA in the health sector was extensively described and related to the case study investigated in this research;
- DEA and network DEA's features were acknowledged in the interest of these methodologies being posteriorly compared;
- Slacks-based network DEA, a freshly new efficiency methodology for the healthcare sector, that incorporates the internal connection between hospital divisions, was developed;
- The slacks-based model was successfully employed for both DEA methodologies;
- The three major CUF's hospitals, CUF Infante Santo, CUF Descobertas, and CUF Porto were analysed using two non-parametric models;
- The results of the case study using both methodologies were detailed compared not only for global efficiency but also for partial efficiency;
- A step forward in the direction of detecting the factors involved in hospital inefficiency was accomplished;
- The inclusion of the business point of view in the results' analysis;
- The comparison of these methodologies allowed a better comprehension of the differences between standard and network DEA's results.

6.2 Recommendations

According to the previous section, it is clear that the inclusion of the hospital's internal connections provides a more insightful view of its efficiency. Furthermore, both methodologies identified a general decrease in efficiency from 2017 to 2019. The business analysis, at the end of the previous sections, allowed to exclude unsuitable justifications for inefficiency. For instance, in 2018 there is an increase in hospital's costs which are associated with FTEs and innovation essential to keep track of progress. Therefore, after calibrating the inefficiency sources, it is possible to formulate the following recommendations. Private healthcare sector's costs, according to CUF's advisor, are mostly associated with medical staff (Bert, 2013). However, the inefficiency does not result from the increase of medical staff but because the increase is not directly correlated with an increase in clients and the hospital's activity.

Accordingly, the first recommendation results (1) there should be a goal-oriented remuneration, where more activity of excellence is encouraged. Note that, this activity must take into consideration not only the quantity but also the quality of performance. The second recommendation arises in alignment with the present dissertation (2) investments in innovative methodologies to measure the hospital's efficiency should be a priority to optimize future decisions based on previous accomplishments.

6.3 Limitations

As far as limitations are concerned, the major one consists in the heterogeneity regarding variables quantity for the different divisions. In other words, the different quantities of internal variables may have resulted in the amplification of dissimilarities among methodologies. For instance, Division 2, which presented the biggest number of internal variables also demonstrated the biggest difference, regarding the identification of efficient DMUs, between standard and network DEA.

Additionally, finding the appropriated variables is normally difficult, as point by (Ozcan, 2008). However, the inclusion of environmental determinants, detailed in Subsection 5.5, such as the legionella outbreak in 2018, provided a major insightful into understanding the causes of inefficiency.

6.4 Future work

As future research, it would be graceful to overcome the limitations previously presented by homogenizing the number of variables used in each division under investigation. Through analysis of the efficiency scores obtained in the next researches, it would facilitate the comprehension of the differences between the application of standard and network DEA. In other words, it is fundamental to study of the impact of specific connections in the determination of global efficiency.

Hereafter, in an academia standpoint, it would be interesting to continue the comparison of efficiency methodologies. On one hand, deepen the investigation of the advantages and disadvantages of the incorporation of internal connections in efficiency analysis. For example, by applying different models than the one developed at the present dissertation to compare standard and network DEA. On the other hand, it would be enlightening to apply the same technique to the public healthcare sector and compare the results obtained. As well as extending the network DEA to a dynamic setting and meta- or partial frontiers capable of adjusting when facing environmental variations (Lobo et al., 2016; Yu and Chen, 2020).

To conclude it would be an exquisite honour to witness the introduction of the hospital's holistic efficiency analysis. The incorporation of internal connections would benefit the decision-making process, as well as comprehension of the interconnectivity of business and social sectors to overcome real-world challenges.

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A

Annexed tables

Table A.1: Review of methodological issues of studies found about hospital efficiency

Authors	Description / Methodology	Variables	Main Conclusions
Stefko et al. (2018)	Evaluate healthcare technical efficiency in individual regions and quantify the basic regional disparities and discrepancies DEA	Inputs: (1) Nr of beds, (2) Nr of medical staff, (3) Nr of all medical equipment, (4) Nr of magnetic resonance (MR) devices, (5) Nr of computed tomography (CT) Outputs: (1) Use of beds, (2) Average nursing time	The regions that had low values of the variables over time achieved a high degree of efficiency and vice versa, Gradual addition of variables did not have a significant impact on the overall estimated efficiency
Marinho (2001)	Compare DEA methodology with performance indicators, in public and private hospital from Brazil DEA	Inputs: (1) Nr of beds, (2) Nr of staff, (3) Nr of doctors, Outputs: (1) Nr of inpatients, (2) Nr of outpatients, (3) Nr of patients treated	DEA has a more consistent result, Efficiency ranges from 15.89 % to 310.5%
Herr (2008)	Investigate both technical and cost efficiency of more than 1500 German general hospitals Cross section SFA	Inputs: (1) Nr of beds, (2) Nr of doctors, (3) Nr of nurses, (4) Nr of other staff, (5) Price of doctors, (6) Price of the nurses, (7) Price of other staff (8) Bed price Outputs: (1) Total adjusted costs, (2) Cases, (3) Weighted cases	Inef[U+FB01]ciencies are present in the German hospital market; It also shown that private and non-pro[U+FB01]t ownership are associated with both lower cost ef[U+FB01]ciency and lower technical ef[U+FB01]ciency when compared with public ownership
Sommersguter-Reichmann and Stepan (2015)	Use of a non-oriented efficiency measure to identify improvement potential inputs and outputs of Austrian hospitals inpatients. Non-oriented super-efficieny measure and the bootstarp algorithm, DEA	Inputs: (1) Nr of full-time equivalent employees, (2) Nr of hospital beds, (3) Expenses for durables and consumables. Outputs: (1) Medical procedures (MEL), (2) Principal diagnosis (HDG)	Quite satiSFActory level of average efficiency scores across the years, Significant efficiency differences between the standard and the extended care level. However, only significant between public hospitals and not between private NFP hospitals.

Table A.1: Review of methodological issues of studies found about hospital efficiency

Authors	Description / Methodology	Variables	Main Conclusions
Tiemann and Schreyögg (2009)	Evaluate the efficiency of public, private for-profit and private non-profit hospitals in Germany Bootstrapped (DEA)	Inputs: (1) Amount spent on supplies per year, (2) Nr of full-time equivalents, (2.1) Clinical staff, (2.2) Nursing staff, (2.3) Medical-technical staff, (2.4) Administrative staff and (2.5) Other staff members Outputs: (1) Nr of treated inpatients, (2) Mortality rate per year	Public hospitals performed significantly better than their private for-profit and non-profit couterparts, Positive association bettween hospital size and efficiency, Competitive pressure has a significant negative impact on hospital efficiency
Campanella et al. (2016)	Methodological framework useful for investigating technical efficiency of hospital care DEA	Inputs (per patient admitted): (1) Nr of beds, (2) Nr of medical doctors, (3) Nr of nurses Outputs: 30-day risk-adjusted mortality for (1) Acute myocardial infarction, (2) Congestive heart failure, (3) Pneumonia	Italian public hospitals had an efficiency score of 77% , Hospital efficiency is likely affected by a various Nr of exogenous variables, The north of Italy, in a region with fiscal autonomy, with a higher public and a lower private expenditure on health as percentage of GDP.
Chitnis and Mishra (2019)	Assess the performance efficiency of Indian private hospitals DEA and super-efficiency DEA	Input: (1) Total capital, (2) Gross fixed assets, (3) Energy Expenses, (4) Compensation to Employees Output: (1) Total Income	Seven of the twenty-five hospitals are efficient and Fortis Hospital Ltd emerges as the super-efficient hospital, Poor utilization of resources is proved to be the major reason for the low performance of hospitals in India
Tiemann and Schreyögg (2012)	Effects of privatization on hospital efficiency in Germany Bootstrapped DEA	Inputs: (1) Amount spent on supplies per year, (2) Nr of full-time equivalents (FTE), (2.1) Physicians , (2.2) Nursing staff, (2.3) Other clinical staff, (2.4) Administrative staff, (2.5) Other non-clinical staff Outputs: (1) Nr of inpatient cases, (2) Mortality rate per year	Increase in efficiency after a conversion to private for-profit status appeared to be permanent. Also shown that the efficiency gains after a conversion to private for-profit status were achieved through substantial decreases in staffing ratios .

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Authors	Description / Methodology	Variables	Main Conclusions
Hsiao et al. (2018)	Evaluate the efficiency of public and private hospitals and investigate the influence of ownership and goals DEA	Inputs: (1) Nr of beds, (2) Nr of administrative staff, (3) Nr of doctors, (4) Nr of nurses, (5) Nr of other medical staff Outputs: (1) Nr of inpatients, (2) Nr of outpatients, (3) Annual emmergency patient visits, (4) Income of inpatients, (5) Income of outpatient	Private and local hospitals are shown to be more efficient than public hospitals, Less market competition and higher average length of stay can increase the efficiency of hospitals
Khania et al. (2012)	Measurement of the relative efficiencies of nine hospitals in the province of Ilam, Iran Output oriented DEA	Inputs: (1) Specialist, (2) Physicians, (3) Technicians, (4) Others Outputs: (1) Surgery operations, (2) Hospital patients, (3)Radiography	For the traditional DEA, four out of the nine hospital were efficient
Kawaguchi et al. (2014)	Evaluation of the policy effect of the reform of Japan's municipal hospitals, focused on efficiency improvements Dynamic Network DEA	Inputs of division 1: (1) Nr of administration officers, (2) Nr of maintenance officers, (3) Cost for financial arrangement, (4) Municipal subsidies to cover deficits, (5) Medical Expenses. Inputs of division 2: (1) Nr of doctors, (2) Nr of nurses, (3) Nr of assistant nurses, (4) Nr of medical technologists Outputs of division 1: (1) Medical Income. Outputs of division 2 (per operation day): (1) Nr of inpatients, (2) Nr of outpatients, (3) Nr of beds in emergency units	No strong correlation between the efficiency scores of the administration division (1) and medical-examination division (2). The ratio of sample hospitals where both divisions improved their efficiency score was 16.96%. The ratio of sample hospitals where the direction of change differed for the two divisions was 24.12%.

Table A.1: Review of methodological issues of studies found about hospital efficiency

Authors	Description / Methodology	Variables	Main Conclusions
Ozcan and Khushalani (2016)	Examination of the change in efficiency of health care systems of 34 OECD countries between 2000 and 2012 Dynamic Network DEA	Inputs of model 1: (1) Smoking consumption, (2) Alcohol consumption, (3) Obesity, (4) Expenses in public health Inputs of model 2: (1) Employees, (2) Hospital beds, (3) Medical technology Outputs of model 1: (1) Life expectancy at birth - Male and Female Outputs of model 2: (1) Hospital discharges, (2) Consultation	Countries that underwent reform showed more improvement in the efficiency of the overall health system and both divisions as compared to the countries that did not. Improvement in the overall health system efficiency was driven by better improvement in the efficiency of the public health system as compared to the medical care system for four of the six countries.
Khushalani and Ozcan (2017)	Analysis on the efficiency of producing quality in hospitals between 2009 and 2013 Dynamic Network DEA	Inputs of model 1: (1) Nr of beds, (2) Total Nr of non-physician, (3) Total operating expenses per bed Input of model 2: (1) Nursing time, (2) Technology services Outputs of model 1: (1) Case-mix adjusted discharges, (2) Total nr of surgeries, (3) Nr of visist, (4) Nr of outpatients Outputs of model 2: (1) Percentage of inpatients who said they would definitely recommend the hospital, (2) Percentage of inpatients who gave the hospital an overall rating of nine and above out of ten	Hospital efficiency improved significantly between 2009 and 2013. Improvement in the efficiency is more likely to be in non-teaching and rural hospitals

Table A.1: Review of methodological issues of studies found about hospital efficiency

Authors	Description / Methodology	Variables	Main Conclusions
Grosskopf et al. (2001)	Comparison of 236 teaching hospitals and 556 non-teaching hospitals in terms of their provision of patient services, operating in the US in 1994. DEA	Inputs: (1) Nr of beds, (2) Nr of physicians with staffing privileges, (3) Nr the medical interns and residents, (4) Nr of registered nurses, (5) Nr of licensed practical nurses, (6) Nr of other labor Outputs: (1) Nr of inpatient, (2) Non-surgical patients treated, (3) Nr of inpatient surgeries, (4) Nr of outpatient surgeries, (5) Nr of emergency room visits, (6) Nr of outpatient visits	The results suggest that only about 10% (23/236) of the teaching hospitals can effectively “compete” with non-teaching hospitals based on the provision of patient services.
Gruga and Nath (2001)	In this study, it was investigated the impact of ownership, size, and location on the relative technical ef[U+FB01]ciency of community hospitals in Ontario, which has a single payer system. DEA	Inputs: (1) Nr of nurses (FTE), (2) Nr of ancillary staff (FTE), (3) Nr of administrations staff (FTE), (4) Services and supplies (including drugs and medical-surgical supplies), (5) Nr of staffed beds Outputs: (1) Inpatients cases weighted by Resources Intensity Weights (RIW), (2) Weighted outpatient visits, (3) Long-term days of care	No significant differences in efficiency across ownership types (government, religious or secular non-profit). Nor significant differences in efficiency by size or location were found. The study suggest that model formulation and differences in payer mix across types of hospitals in the US have a strong influence on the measurement of the hospital ownership–efficiency relationship.
Biørn et al. (2003)	Analysis of the prediction, which assumed that hospital efficiency will increase because the benefit from cost-reducing efforts in terms of Nr of treated patients is increased under activity-based financing compared with global budgets. The prediction is testedusing a panel data set from the period 1992-2000. DEA	Inputs: (1) Physician (FTE), (2) Other staff (FTE), (3) Medical expenses, (4) Total running expenses Outputs: (1) Nr of input patients (DRGs), (2) Nr of output patients weighted by the fee paid	The introduction of ABF has improved efficiency when measured as technical efficiency according to DEA analysis. Contrary to our prediction, the result is less uniform with respect to the effect on cost-efficiency.

Table A.1: Review of methodological issues of studies found about hospital efficiency

Authors	Description / Methodology	Variables	Main Conclusions
Maniadakis and Thanassoulis (2004)	Application of a productivity index used when producers are cost minimisers and input princes are known. Productivity is change is decomposed into overall efficiency and cost technical change. DEA	Inputs: (1) Nr of doctors, (2) Nr of nurses, (3) Other staff, (4) Doctors salary, (5) Nurses salary, (6) Other staff salary Output: (1) Nr of inpatients, (2) Clinical examination, (3) Laboratory tests	A productivity index that accounts not only technical efficiency and technological changes but also allocative efficiency and for the effects of input price changes is developed. It allows the identification of the root sources of productivity changes and to derive information on decision making when input prices are available.
Sahin et al. (2011)	Explore the operational performance of the Ministry of Health's general public hospitals following the implementation of the Health Transformation Program in Turkey, during the period of 2005-2008. acDEA	Inputs: (1) Nr of beds, (2) Nr of physicians (FTE), (3) Nr of nurses (FTE), (4) Other staff (FTE), (5) Operational expenses Outputs: (1) Nr of outpatients, (2) Nr of inpatients, (3) Total nr of surgeries	The result show that the operational performance of these hospitals regardless of the attributes present a common tendency that the performance of 2005–2007 progressed as compared to the previous year, while that of 2008 has regressed as compared to 2007.
Hu et al. (2012)	Analysis on regional hospital efficiency in China during the 2002–2008 period, especially the health insurance reform of New Rural Cooperative Medical System (NRCMS) impacts on efficiency DEA	Inputs: (1) Nr of doctors, (2) Nr of technicians, (3) Other staff, (4) Nr of hospital beds, (5) Medical Equipment Ouputs: (1) Nr of outpatients, (2) Nr of inpatients, (3) Patients mortality	The regression analysis on examining determinants of efficiency suggests that a higher proportion of for-profit hospital and high quality hospital is helpful to enhance technical efficiency. Negative relationship between government subsidy and efficiency for coastal regions. Medical reform of NRCMS overall has a significant efficiency-enhancing effect, particularly for non-coastal region

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Authors	Description / Methodology	Variables	Main Conclusions
Caballer-Tarazona et al. (2010)	Analyse efficiency in three healthcare service units in Valencian hospitals to establish appropriate guidelines for efficiency performance DEA and efficiency indicators (Incomes/doctors and Interventions/doctors)	Inputs: (1) Nr of doctors, (2) Nr of beds Outputs: (1) Nr of admissions weighted by case-mix, (2) Nr of consultations, (3) Nr of successive consultations, (4) Nr of surgical interventions	The efficiency analyzes is using the DEA Model is considered more useful when studying the efficiency of each service separately instead of studying the overall efficiency of a given hospital. The levels of efficiency of the services analysed were above the mean
Bilsel and Davutyan (2014)	Analysis of the operational performance of 202 Turkish rural general hospitals. DEA with directional distance approach,	Inputs: (1) Nr of beds, (2) Nr of Specialist (FTE), (3) Nr of general practitionares (FTE), (4) Nr of nurses (FTE), (5) Other staff (FTE), (6) Operational expenses Outputs: (1) Nr of outpatients, (2) Nr of inpatients, (3) Nr of surgeries, (4) Nr of DEAths per surgery	It was shown that "reducing mortality" involves sacri[U+FB01]cing some outputs. This is a trade off that holds at the potential output level. In particular, it was demonstrated the relative scarcity of nurses is linked to output congestion. For a more precise analysis it requires introducing DRG based case mix weighting and objective measures of care quality
Barnum et al. (2011)	Comparision of DEA theory's requirement that inputs (outputs) be substitutable, and the ubiquitous use of nonsubstitutable inputs and outputs in DEA applications to hospitals. DEA	Inputs: (1) Nr of beds, (2) Nr of FTE employees Outputs: (1) Nr of inpatients, (2) Nr of outpatients	DEA substantially overestimated the hospitals' efficiency on the average, and reported many inefficient hospitals to be efficient. These results suggest that conventional DEA models should not be used to estimate the efficiency of hospitals unless there is empirical evidence that the inputs (outputs) are substitutable

Table A.1: Review of methodological issues of studies found about hospital efficiency

Authors	Description / Methodology	Variables	Main Conclusions
Vitikainen et al. (2009)	Analyse the influence of the chosen output and case-mix measures in the efficiency results. Examining the robustness of efficiency results to theses factors. DEA	Inputs. (1) Operational costs (labour, material and depreciated capital costs) Outputs 1st Approach (episodes): (1) Nr of inpatients, (2) Nr of outpatients, (3) Nr of emergencies Outputs 2nd Approach (admissions and visits): (1) Nr of admissions, (2) Surgery per day, (3) Scheduled visits, (4) Emergency visists	Efficiency estimates were found not to be highly sensitive to the choice between episode and activity description of output, but more to the choice of DRG grouping system.
Harrison et al. (2015)	This study evaluates the efficiency of small and large for-profit hospitals. For this study, small for-profit hospitals are 35 beds or less which is consistent with the Federal designation of critical access hospitals (CAH). DEA	Inputs: (1) Operating expenses, (2) Nr of beds, (3) Nr of staff (FTE) Outputs: (1) Inpatient Days, (2) Outpatient Visits, (3) Surgical Procedures	Results indicate overall efficiency in small for-profit hospitals was 60% in 2013. In contrast, the overall efficiency in large for-profit hospitals was 71% in 2013. This documents for-profit hospitals' overall efficiency increases with greater size.
Carey (2003)	This paper estimates a stochastic frontier cost function to test for inefficiency differences among system hospitals having common strategic and/or structural characteristics. SFA	Inputs: (1) Average annual salary, (2) Nr of beds, Outputs: (1) Nr of admissions, (2) Adjusted patients per day, (3) Case-mix index	This suggests efficiency benefits from organization of physician and insurance activities at the system level, with discretion over the array of service offerings left to individual members.
Herr et al. (2011)	This paper investigates the cost and profit efficiency of German hospitals and their variation with ownership type. It was motivated by the empirical [U+FB01]nding that private (for-pro[U+FB01]t) hospitals – having been shown to be less cost ef[U+FB01]cient in the past – on average earn higher pro[U+FB01]ts than public hospital. SFA	Inputs: (1) Nr of beds, (2) Nr of doctors, (3) Nr of nurses, (4) Other staff, (5) Price of doctors, (6) Price of the nurses, (7) Price of other staff (8) Bed price, (9) Medical requirements per case, (10) Administritive costs, (11) Other materials costs Outputs: (1) Total adjusted costs, (2) Weighted cases	The results show no significant differences in cost efficiency but higher pro[U+FB01]t efficiency of private than of publicly owned hospitals. However, under the assumption that outcome quality does not differ between ownership types, private hospitals are performing better in [U+FB01]nancing investments without recurring on taxpayers' money

Table A.1: Review of methodological issues of studies found about hospital efficiency

Authors	Description / Methodology	Variables	Main Conclusions
Nayar and Ozcan (2008)	Using a sample of Virginia hospitals, performance measures of quality were examined as they related to technical efficiency. DEA	Inputs: (1) Nr of beds, (2) Operational expenses, (3) Nr total of staff, (4) Total assets Outputs: (1) Nr of inpatients, (2) Nr of outpatients, (3) FTE	The study found that the technically efficient hospitals were performing well as far as quality measures were concerned. Some of the technically inefficient hospitals were also performing well with respect to quality.
Ng (2011)	Based on data from 2004-2008,the present study aims at providing empirical evidence of inefficiency of hospitals in China. DEA	Inputs per year: (1) Nr of doctors, (2) Nr of nurses, (3) Nr of pharmacists, (4) Other staff, (5) Nr of beds Outputs per year: (1) Nr of inpatients treated, (2) Nr of outpatients treated	Echoing the unnecessary care, over-prescription of drugs and the adoption of hightech treatments since the implementation of health care reforms, the sampled hospitals were found quite inefficient and pure technical inefficiency played a dominant role in driving the inefficiency of hospitals. Hospitals had experienced productivity growth between 2004 and 2008.
Linna et al. (2006)	In this study, cost efficiency in Finnish and Norwegian hospitals was compared using national discharge data and identical definitions for cost and output measures DEA	Inputs: (1) Net hospital operating costs Outputs: (1) Admissions grouped according to DRGs, (2) Outpatient visits, (3) Outpatient day care, (4) Nr of inpatient days	According to preliminary results there was more variation in cost efficiency among Finnish hospitals, and the average level of cost efficiency was 17-25% lower in Norwegian hospitals.
Mateus et al. (2015)	The objectives of this study is to assess and compare hospital efficiency levels within and between countries. SFA with both cross-sectional and panel data.	Inputs: (1) Nr of employees, (2) Nr of physicians, (2) Nr of nurces, (3) Nr of beds Outputs: (1) Inpatients weighted hospital discharges	The likelihood ratio test reveals that in cross-sectional data SFA is not statistically different from OLS in Portuguese data, while SFA and OLS estimates are statistically different for Spanish, Slovenian and English data. Panel data are preferred over cross-section analysis because results are more robust. For all countries except Slovenia, beds and employees are relevant inputs for the production process

Table A.2: DMU detailed information according to the year, hospital, and month under evaluation

Year	DMU Description	DMU n	Year	DMU Description	DMU n	Year	DMU Description	DMU n
2017	HCIS_Jan	DMU 1	2018	HCIS_Jan	DMU 37	2019	HCIS_Jan	DMU 73
	HCIS_Feb	DMU 2		HCIS_Feb	DMU 38		HCIS_Feb	DMU 74
	HCIS_Mar	DMU 3		HCIS_Mar	DMU 39		HCIS_Mar	DMU 75
	HCIS_Apr	DMU 4		HCIS_Apr	DMU 40		HCIS_Apr	DMU 76
	HCIS_May	DMU 5		HCIS_May	DMU 41		HCIS_May	DMU 77
	HCIS_Jun	DMU 6		HCIS_Jun	DMU 42		HCIS_Jun	DMU 78
	HCIS_Jul	DMU 7		HCIS_Jul	DMU 43		HCIS_Jul	DMU 79
	HCIS_Aug	DMU 8		HCIS_Aug	DMU 44		HCIS_Aug	DMU 80
	HCIS_Sep	DMU 9		HCIS_Sep	DMU 45		HCIS_Sep	DMU 81
	HCIS_Oct	DMU 10		HCIS_Oct	DMU 46		HCIS_Oct	DMU 82
	HCIS_Nov	DMU 11		HCIS_Nov	DMU 47		HCIS_Nov	DMU 83
	HCIS_Dec	DMU 12		HCIS_Dec	DMU 48		HCIS_Dec	DMU 84
	HDSC_Jan	DMU 13		HDSC_Jan	DMU 49		HDSC_Jan	DMU 85
	HDSC_Feb	DMU 14		HDSC_Feb	DMU 50		HDSC_Feb	DMU 86
	HDSC_Mar	DMU 15		HDSC_Mar	DMU 51		HDSC_Mar	DMU 87
	HDSC_Apr	DMU 16		HDSC_Apr	DMU 52		HDSC_Apr	DMU 88
	HDSC_May	DMU 17		HDSC_May	DMU 53		HDSC_May	DMU 89
	HDSC_Jun	DMU 18		HDSC_Jun	DMU 54		HDSC_Jun	DMU 90
	HDSC_Jul	DMU 19		HDSC_Jul	DMU 55		HDSC_Jul	DMU 91
	HDSC_Aug	DMU 20		HDSC_Aug	DMU 56		HDSC_Aug	DMU 92
	HDSC_Sep	DMU 21		HDSC_Sep	DMU 57		HDSC_Sep	DMU 93
	HDSC_Oct	DMU 22		HDSC_Oct	DMU 58		HDSC_Oct	DMU 94
	HDSC_Nov	DMU 23		HDSC_Nov	DMU 59		HDSC_Nov	DMU 95
	HDSC_Dec	DMU 24		HDSC_Dec	DMU 60		HDSC_Dec	DMU 96
	HPRT_Jan	DMU 25		HPRT_Jan	DMU 61		HPRT_Jan	DMU 97
	HPRT_Feb	DMU 26		HPRT_Feb	DMU 62		HPRT_Feb	DMU 98
	HPRT_Mar	DMU 27		HPRT_Mar	DMU 63		HPRT_Mar	DMU 99
	HPRT_Apr	DMU 28		HPRT_Apr	DMU 64		HPRT_Apr	DMU 100
	HPRT_May	DMU 29		HPRT_May	DMU 65		HPRT_May	DMU 101
	HPRT_Jun	DMU 30		HPRT_Jun	DMU 66		HPRT_Jun	DMU 102
	HPRT_Jul	DMU 31		HPRT_Jul	DMU 67		HPRT_Jul	DMU 103
	HPRT_Aug	DMU 32		HPRT_Aug	DMU 68		HPRT_Aug	DMU 104
	HPRT_Sep	DMU 33		HPRT_Sep	DMU 69		HPRT_Sep	DMU 105
	HPRT_Oct	DMU 34		HPRT_Oct	DMU 70		HPRT_Oct	DMU 106
	HPRT_Nov	DMU 35		HPRT_Nov	DMU 71		HPRT_Nov	DMU 107
	HPRT_Dec	DMU 36		HPRT_Dec	DMU 72		HPRT_Dec	DMU 108

Table A.3: Description of DMUs that are only efficient for one of the methodologies under analysis.

Efficient DMUs only for standard DEA	**Efficient DMUs only for network DEA**
HCIS_Jan2017, HCIS_Feb2017, HCIS_Jul2017, HCIS_Sep2017, HDSC_Mar2017, HDSC_Dez2017, HPRT_Jan2017, HPRT_Feb2017, HPRT_Mar2017, HPRT_Apr2017, HPRT_May2017, HPRT_Jun2017, HPRT_Jul2017, HPRT_Sep2017, HPRT_Nov2017, HPRT_Dez2017, HDSC_Mar2018, HDSC_Ago2018, HDSC_Nov2018, HDSC_Dez2018, HPRT_Jan2018, HPRT_Feb2018, HPRT_Mar2018, HPRT_Apr2018, HPRT_May2018, HPRT_Jun2018, HPRT_Jul2018, HPRT_Ago2018, HPRT_Nov2018, HPRT_Dez2018, HCIS_Jan2019, HCIS_Feb2019, HCIS_Mar2019, HCIS_Apr2019, HCIS_May2019, HCIS_Jun2019, HCIS_Jul2019, HCIS_Ago2019, HCIS_Nov2019, HCIS_Dez2019, HDSC_Jun2019, HPRT_Jan2019, HPRT_Feb2019, HPRT_Mar2019, HPRT_Apr2019, HPRT_May2019, HPRT_Jul2019, HPRT_Ago2019, HPRT_Oct2019, HPRT_Nov2018, HPRT_Dez2018,	HCIS_Oct2017, HDSC_Jan2017, HDSC_May2017, HDSC_Oct2017, HDSC_Nov2017, HICS_Jan2018, HICS_Jun2018, HDSC_Jan2018